

STRESS ANALYSIS, BUCKLING ANALYSIS AND
OPTIMUM PROPORTIONS OF AN ISOTROPIC
WEB-STIFFENED SANDWICH CYLINDRICAL
SHELL UNDER HYDROSTATIC PRESSURE

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UNDER HYDROSTATIC PRESSURE

by

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(1961)

SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE
DEGREES OF NAVAL ENGINEER
AND MASTER OF SCIENCE IN
OCEAN ENGINEERING

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
May, 1970

These notes

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2.

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Submitted to the Department of Naval Architecture and Marine Engineering on May 21, 1970, in partial fulfillment of the requirements for the degree of Naval Engineer in Naval Architecture and Marine Engineering and Master of Science in Ocean Engineering.

ABSTRACT

Web-stiffened sandwich cylindrical shells consisting of two concentric cylindrical shells joined and stiffened by annular rings (webs) were considered in the investigation. The purpose of this investigation was to study the stress distribution and buckling modes, and to determine the scantlings and configuration of optimum proportions.

The scantlings and configuration of optimum proportions was defined as that with the lowest weight to displacement ratio at a given loading of hydrostatic pressure.

The scantlings (outer shell thickness, inner shell thickness, web thickness, web depth (separation of shells) and configuration (web spacing) were systematically and independently varied and the effect upon failure mode pressures, and the weight to displacement ratio was determined. The results were plotted graphically.

The form of stress and buckling analysis results follow closely those of a ring-stiffened cylinder. Geometric similitude was determined to exist. The scantlings and configuration of optimum proportions were found to be a function of the loadings.

Web-core sandwich configurations possess structural efficiencies on the order of 20% higher than those of conventional ring-stiffened construction for loadings on the order of 1000 psi (2225 feet) and these diminish with depth to about 5% higher for loadings on the order of 4000 psi (8900 feet).

The real advantage of the web-stiffened sandwich construction lies in the use of thinner plating with all its superior metallurgical and fabrication qualities.

Thesis Supervisor: Professor J. H. Evans
Title: Professor of Naval Architecture

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NOTATION

b	Web thickness
c	$(E_s/E_T) - 1$
C	Web constant
D	Bending, rigidity, $Et^3/12(1-\nu^2)$
D_M	Mean diameter of shells
d	RRO-RRI
E	Young's modulus
E_s	Secant modulus
E_T	Tangent modulus
f	Stress ratio, σ_x/σ_ϕ
g_o, g_i	Edge coefficients for outside and inside shells
$g_{oo}, g_{oi}, g_{io}, g_{ii}$ (M,N,P,Q)	Lamé deflection coefficients for the annular webs
h	$R_o - R_i$
k_i	Stress intensity per pound of pressure ($K_x = \sigma_x/p$)
L	Unsupported length of cylinder
L_B	Length between bulkheads
m, n, k	Integers
N_x, N_ϕ	Longitudinal and circumferential forces per unit length
x, ϕ , r	Longitudinal, circumferential, and radial coordinates

B_{io}, F_{io}, G_{io}	Hyperbolic and trigonometric stress parameters for outside shell
B_{ii}, F_{ii}, G_{ii}	Hyperbolic and trigonometric stress parameters for inside shell
M_x	Longitudinal moment per unit length
p	External hydrostatic pressure
g_o, d_o, f_o	Force coefficients for outside shell
g_i, d_i	Force coefficients for inside shell
$H_o, H_i (H_{HO}, H_{HI})$	Total radial forces on the outer and inner surface of webs per unit length
$N_o, N_i (V_o, V_i)$	Longitudinal forces on outside and inside shells per unit length
$Q_o, Q_i (H_o, H_i)$	Radial forces on outside and inside shells per unit length
()	Same quantity used in general theory and computer programs
Q_x	Radial force per unit length
R	Radius
R_o, R_i	Mean radius of outside and inside shells
RRO, RRI	Extreme radius of outer fiber of outside and inside shells
RR	RRI/RRO
S	Mean distance between webs
t	Thickness of shell
T_o, TO	Outside shell thickness
T_i, TI	Inside shell thickness

u, v, w	Longitudinal circumferential and radial deflections
W_{os}, W_{is}	Radial deflection of outside and inside shell
W_{ow}, W_{iw}	Radial deflection of the outer and inner surfaces of the web
Z	Radial bending rigidity, $E/(1-\nu^2)$
$\epsilon_x, \epsilon_\phi, \epsilon_r$	Longitudinal, circumferential, and radial strains
$\sigma_x, \sigma_\phi, \sigma_r$	Longitudinal, circumferential, and radial stresses
θ_o, θ_i	Shell flexibility parameters for outside and inside shells, $[3(1-\nu^2)]^{1/4} [L/(Rt)]^{1/2}$
ψ	Bending stress coefficient, $[(1-\nu^2)/3]^{1/2}$
ν	Poisson's ratio for isotropic materials
σ_i	Stress intensity, Huber-Hencky, Von Mises, $[(\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2)^{1/2}]$
σ_y	Yield stress
σ_{ijkl} (σ_{iKL})	Stress location
$i = x, \phi, r$	direction
$j = F, M$	shell (web-shell juncture, midbay)
$K = O, I, W$	outside shell, inside shell, web
$L = O, I, M$	fiber (outer, inner, membrane)
(j_iKL)	i.e., MXOI = midbay, longitudinal, outside shell, inner fiber
P	Yield failure at shell midbay

PF	Yield failure at web-shell juncture
PWS	Yield failure at web outer fiber
PPC1	Axisymmetric shell buckling
PPC2	Asymmetric shell buckling
PPC3	General instability
PPC4	Web, ring buckling
PPC5	Web, edge compression buckling

INTRODUCTION

One of the more promising structural concepts for the design and fabrication of cylindrical pressure hull structures is the sandwich concept. Structural engineers in the aircraft industry have long recognized and taken advantage of the favorable strength-weight characteristics of sandwich-type construction. In studying the literature, however, it has been found that the loading conditions encountered in these applications have dictated sandwich structural arrangements that would be of no direct use in the design of pressure hulls for submersibles.

The demands of hydrostatic pressure loading are such that in order to exploit the sandwich concept for pressure hull construction a core comprised of elements which are compression resistant as well as shear resistant is sought. The major difference is that membrane loads are predominant in hydrospace applications, whereas in aerospace, the bending and shear type loads are of prime concern.

Experimental programs and analytical studies have been going on concurrently in order to develop rational formulas based on thin-shell theory for predicting the static structural response of these types of structures. In reference 10 Pulos presents an analysis of the axisymmetric elastic

deformations and stresses in a web-stiffened sandwich cylinder under hydrostatic pressure. Raetz presents a similar analysis for the toroidal tube-stiffened sandwich cylinder.¹¹ In references 12 and 14 Nott presents a simplified stress and strain analysis.

Nott's equations were used and programmed for the stress analysis of the web-stiffened structure.

In reference 22 the axisymmetric and asymmetric buckling equations are developed. In references 15 and 23 an expression for the general instability is determined.

The above equations are utilized, with others derived for web stability, to study the parametric nature of web-core sandwich structure and to determine optimum proportions.

Optimum design must be based on rational considerations of inelastic behavior, however, and not on the "one horse shay" concept based on elastic considerations of instability. The "ignorance factors" can then be representative of the variability introduced by certain intangibles which are not easily considered in a theory.

Some of the intangibles which influence static strength and complicate the problem so that appropriate design formulas cannot be derived on purely theoretical grounds are those inherent in the fabrication process itself:

- (1) Initial stress.
- (2) Residual stress.
- (3) Imperfect circularity.
- (4) Non-isotropic or non-homogeneity of material.
- (5) Actual boundary conditions at stiffening rings.

Therefore, in view of these complications, the development of satisfactory design criteria must first be predicted on rigorous mathematical theory with its concomitant idealizations, and then, empirical factors derived from test data can be introduced to "adjust" the theories to take account of these many variables which could not or were not considered.

Thus an inelastic analysis of all instability modes is used.

GENERAL THEORY

1. Stress Analysis

A theoretical analysis of the axisymmetric elastic deformations and stresses in a web-stiffened sandwich cylindrical shell structure under external hydrostatic pressure is presented in Appendix A. The solution is based on the use of edge coefficients for plate and shell elements of finite length, and includes the computation of the edge forces and moments arising at the common junctions of these elements.

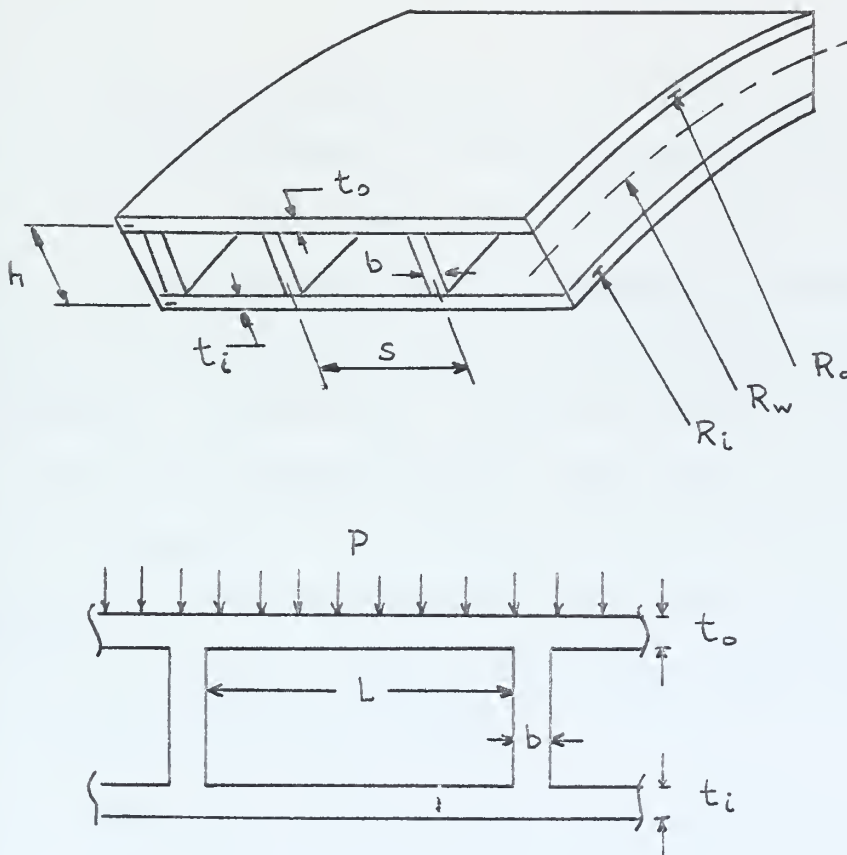
Equations are given for computing numerically the longitudinal and circumferential stresses in the two coaxial cylindrical shells and the radial and tangential stresses in the web stiffeners between the two shells.

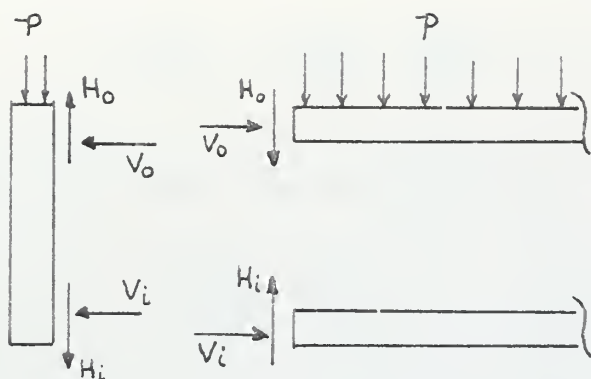
To evaluate the structural strength of cylindrical sandwich shells, the locations and magnitudes of maximum stresses must be determined. Rigorous analyses for stress distributions in sandwich shells loaded under external hydrostatic pressure were carried out by Pulos¹⁰ and Raetz¹¹. These analyses illustrate that the maximum stresses occur in the inner and outer shells at locations next to the annular webs and midway between the two webs that separate the shells. Therefore, the stresses in the inside and outside shells at these two locations are of

interest for the purpose of structural design and evaluation.

The procedure and nomenclature for programming the stress analysis is that of Nott.^{12,14}

a. NOMENCLATURE





b. ASSUMPTIONS

From symmetry considerations it is seen that the edges of the web stiffener, circular annulus, do not undergo any rotation. This stems from the fact that a horizontal tangent or zero-slope condition is assumed to exist at the junctions of the webs with the two cylindrical shells. This assumption implies that the edge moments on each shell at the shell-web juncture balance each other, so that there are no net moments to be resisted by the web. Further, it is assumed that the web elements do not take any axial force due to the axial pressure, but that this is all resisted by the cylindrical shells.

Also inherent in the analysis (see Appendix A) is the neglect of the beam-column effect due to the axial portion of the hydrostatic pressure.

$$D \frac{d^4 w}{dx^4} + N_x \frac{d^2 w}{dx^2} + \frac{Et}{R^2} W = P - \frac{\nu}{R} N_x$$

neglecting the beam-column effect becomes

$$D \frac{d^4 w}{dx^4} + \frac{Et}{R^2} W = P - \frac{\nu}{R} N_x$$

Thus the analysis of the web stiffeners reduces to that of a circular annulus subjected to axisymmetric in-plane radial forces on both its inner and outer boundaries.

On the basis of these assumptions, it is only necessary to derive edge coefficients for an annulus undergoing radial deflection.

The functions M , N , P , and Q are the Lamé deflection coefficients for the annular webs and can be expressed

$$M = \frac{R_w}{b} \left(1 + \frac{d}{2R_w} \right) \left(\frac{2R_w}{d} + \frac{d}{2R_w} - 2\nu \right)$$

$$N = \frac{R_w}{b} \left(1 + \frac{d}{2R_w} \right) \left(\frac{2R_w}{d} + \frac{d}{2R_w} - 2 \right)$$

$$P = \frac{R_w}{b} \left(1 - \frac{d}{2R_w} \right) \left(\frac{2R_w}{d} + \frac{d}{2R_w} + 2 \right)$$

$$Q = \frac{R_w}{b} \left(1 - \frac{d}{2R_w} \right) \left(\frac{2R_w}{d} + \frac{d}{2R_w} + 2\nu \right)$$

and the functions g_o , g_i are the edge coefficients from the outside and inside shells:

$$g_o = - \frac{2R_o^2}{T_o L} F_{10}$$

$$g_i = - \frac{2R_i^2}{T_i L} F_{1i}$$

where

$$F_{10} = \frac{\theta_o}{2} \left[\frac{\sinh \theta_o + \sin \theta_o}{\cosh \theta_o - \cos \theta_o} \right]$$

$$F_{1i} = \frac{\theta_i}{2} \left[\frac{\sinh \theta_i + \sin \theta_i}{\cosh \theta_i - \cos \theta_i} \right]$$

$$\theta_o = \sqrt[4]{3(1-v^2)} \cdot \frac{L}{\sqrt{R_o T_o}}$$

$$\theta_i = \sqrt[4]{3(1-v^2)} \cdot \frac{L}{\sqrt{R_i T_i}}$$

Thus, from Appendix A, the axial and transverse forces (V_o , V_i , H_o and H_i), which act at the intersections of the annular webs and cylinders can be determined.

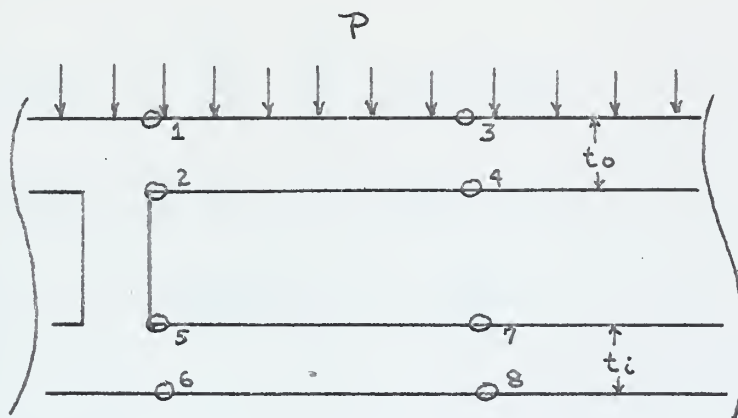
$$V_i = \frac{T_i}{T_o} V_o$$

$$V_o = \frac{R_o/2}{1 + \frac{T_i}{T_o} \cdot \frac{R_i}{R_o}}$$

$$(g_o - M) H_o - N H_i = - \frac{R_o^2}{T_o} \left(1 - \frac{v/2}{1 + \frac{T_i}{T_o} \cdot \frac{R_i}{R_o}} \right) + \frac{b}{2} M$$

$$-P H_o + (g_i - Q) H_i = \frac{v}{2} \frac{R_o^2}{T_i} \left(1 - \frac{1}{1 + \frac{T_i}{T_o} \cdot \frac{R_i}{R_o}} \right) + \frac{b}{2} P$$

And from these forces the maximum stresses in the shells can be determined.



C. MAXIMUM STRESSES

OUTER SHELL

$$1. \quad \sigma_{xfoo} = \left(+ \frac{V_o}{R_o} - \frac{H_o}{L} (B_{40}) \right) \sigma_{uo}$$

$$\sigma_{\phi foo} = \left(+ 1.0 + \frac{H_o}{L} (B_{50}) \right) \sigma_{uo}$$

$$2. \quad \sigma_{xfoi} = \left(+ \frac{V_o}{R_o} + \frac{H_o}{L} (B_{40}) \right) \sigma_{uo}$$

$$\sigma_{\phi foi} = \left(+ 1.0 - \frac{H_o}{L} (B_{60}) \right) \sigma_{uo}$$

$$3. \quad \sigma_{xmoo} = \left(+ \frac{V_o}{R_o} + \frac{H_o}{L} (B_{30}) \right) \sigma_{uo}$$

$$\sigma_{\phi moo} = \left(+ 1.0 - \frac{H_o}{L} (B_{30}) \right) \sigma_{uo}$$

$$4. \quad \sigma_{xmoi} = \left(+ \frac{V_o}{R_o} - \frac{H_o}{L} (B_{30}) \right) \sigma_{uo}$$

$$\sigma_{\phi moi} = \left(+ 1.0 - \frac{H_o}{L} (B_{20}) \right) \sigma_{uo}$$

$$\text{where } \sigma_{uo} = -p \frac{R_o}{T_o}$$

INNER SHELL

$$5. \quad \sigma_{xfio} = \left(+ \frac{V_i}{R_i} - \frac{H_i}{L} (B_{4i}) \right) \sigma_{ui}$$

$$\sigma_{\phi fio} = \left(- \frac{H_i}{L} (B_{5i}) \right) \sigma_{ui}$$

$$6. \quad \sigma_{xfii} = \left(+ \frac{V_i}{R_i} + \frac{H_i}{L} (B_{4i}) \right) \sigma_{ui}$$

$$\sigma_{\phi fii} = \left(- \frac{H_i}{L} (B_{6i}) \right) \sigma_{ui}$$

$$7. \quad \sigma_{xmio} = \left(+ \frac{V_i}{R_i} + \frac{H_i}{L} (B_{3i}) \right) \sigma_{ui}$$

$$\sigma_{\phi mio} = \left(- \frac{H_i}{L} (B_{1i}) \right) \sigma_{ui}$$

$$8. \quad \sigma_{xmii} = \left(+ \frac{V_i}{R_i} - \frac{H_i}{L} (B_{3i}) \right) \sigma_{ui}$$

$$\sigma_{\phi mii} = \left(- \frac{H_i}{L} (B_{2i}) \right) \sigma_{ui}$$

$$\text{where} \quad \sigma_{ui} = -p \frac{R_i}{T_i}$$

and

$$B_{10} = 2 F_{20} - v F_{30}$$

$$B_{20} = 2 F_{20} + v F_{30}$$

$$B_{30} = F_{30}$$

$$B_{40} = F_{40}$$

$$B_{50} = 2 F_{10} + v F_{40}$$

$$B_{60} = 2 F_{10} - v F_{40}$$

$$F_{10} = \frac{\theta_o}{2} \left(\frac{\sinh \theta_o + \sin \theta_o}{\cosh \theta_o - \cos \theta_o} \right)$$

$$F_{20} = \theta_o \left[\frac{\cosh \theta_o/2 \sin \theta_o/2 + \sinh \theta_o/2 \cos \theta_o/2}{\cosh \theta_o - \cos \theta_o} \right]$$

$$F_{30} = \frac{6\theta_o}{\sqrt{3(1-v^2)}} \left[\frac{\cosh \theta_o/2 \sin \theta_o/2 - \sinh \theta_o/2 \cos \theta_o/2}{\cosh \theta_o - \cos \theta_o} \right]$$

$$F_{40} = \frac{3\theta_o}{\sqrt{3(1-\nu^2)}} \frac{\sinh \theta_o - \sin \theta_o}{\cosh \theta_o - \cos \theta_o}$$

$$\theta_o = \sqrt[4]{3(1-\nu^2)} \cdot \frac{L}{\sqrt{R_o T_o}}$$

and likewise for $B_{1i}, B_{2i}, \dots, F_{4i}, \theta_i$ by substituting the inside I for the outside O.

d. WEB STRESSES

From Appendix A the maximum stress in the web

$$\sigma_{rwo} = -p \frac{[(-HHI(RR)^2 + HHO) - (-HHI(RR) + HHO(RR))(RR)]}{b/2 (1 - (RR)^2)}$$

$$\sigma_{\phi wo} = -p \frac{[(-HHI(RR)^2 + HHO) + (-HHI(RR) + HHO(RR))(RR)]}{b/2 (1 - (RR)^2)}$$

where

$$HHO = H_o + b/2$$

$$HHI = H_i$$

$$RR = RRI/RRO$$

$$RRI = R_i - T_i/2$$

$$RRO = R_o + T_o/2$$

2. Buckling Analysis and Failure Modes

The axisymmetric and asymmetric (lobar) buckling equations developed by Lunchick⁴ and Reynolds⁸ for ring-stiffened cylinders are modified in Appendix B for application to web-stiffened sandwich cylinders. They are evaluated for both the inner and outer shells. Reference 22 indicates good agreement between theoretical and experimental collapse pressures.

The general instability equation used has found considerable experimental verification.^{15,23}

The two modes of web buckling are first, a variation of the Foppl or Levy ring buckling (effectively TOKUGAWA) and second, the buckling of an annulus in biaxial edge compression (see Appendix B).

a. FAILURE MODES

1. STRENGTH

Using the Huber-Hencky-Von Mises criterion

$$\sigma_i = \sqrt{\sigma_\phi^2 + \sigma_x^2 - \sigma_\phi \sigma_x}$$

$$\sigma_i = p \sqrt{K_\phi^2 + K_x^2 - K_\phi K_x}$$

$$\therefore p = \frac{\sigma_y}{\sqrt{K_\phi^2 + K_x^2 - K_\phi K_x}}$$

evaluated at the web-shell interface (PF), midbay (P), and outer fiber of web (PWS).

2. AXISYMMETRIC BUCKLING

$$PPC1 = \frac{E_T L^2}{12(1-\nu^2)K_X \pi^2 R^2} \left[\frac{\pi^4 R^2 t^2 (1 + \frac{3c}{4})}{L^4} + \frac{12(1+c)(1-\nu^2)}{(1 + \frac{3c}{4})} \right]$$

where

$$c \equiv E_S/E_T - 1$$

$$\nu \equiv 1/2 - E_S/E(1/2 - \nu_E)$$

3. ASYMMETRIC BUCKLING

$$PPC2 = \frac{\pi^2 E_T f_E}{3K_X \phi (1-\nu^2)} \frac{t}{R} \left(\frac{(Rt)^{1/2}}{L} \right)^2 \left(\frac{1 + \frac{3\phi}{4}c}{3 - 2\phi(1-f_E)} \right)$$

where

$$\phi \approx 1.23 \frac{(Rt)^{1/2}}{L}$$

$$f_E \equiv K_X/K_\phi$$

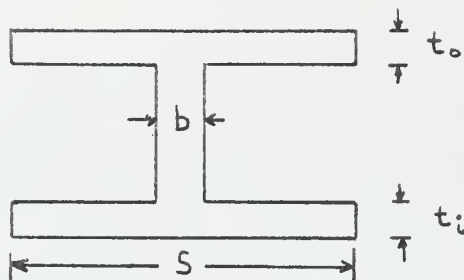
4. GENERAL INSTABILITY

$$PPC3 = \sqrt{E_T \cdot E_S} \cdot \frac{t_T}{R} \left[\frac{\lambda^4}{\left[n^2 - 1 + \frac{\lambda^2}{2} \right] (n^2 + \lambda^2) \left\{ 1 - 3/4 \left(1 - \frac{E_S}{E_T} \right) \frac{\lambda^4}{(n^2 + \lambda^2)^2} \right\}} \right] + \frac{(n^2 - 1) E_T I_{EFF}}{R^3 L}$$

where

$$\lambda = \frac{\pi R}{L_B}$$

I_{eff} for



and

$$t_T = T_O + T_i$$

$$R = R_W$$

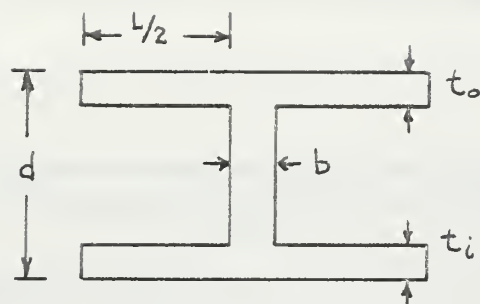
5. WEB BUCKLING

a. Ring Buckling

$$\text{PPC4} = \frac{24 E_T I_{\text{eff}}}{D_m^3 L (1-\nu^2)}$$

b. Edge Compression

$$\text{PPC5} = \frac{4\pi^2 E_T}{12 (1-\nu^2) (K_\phi + 4K_R)} \left(\frac{b}{h} \right)^2$$

I_{eff} :

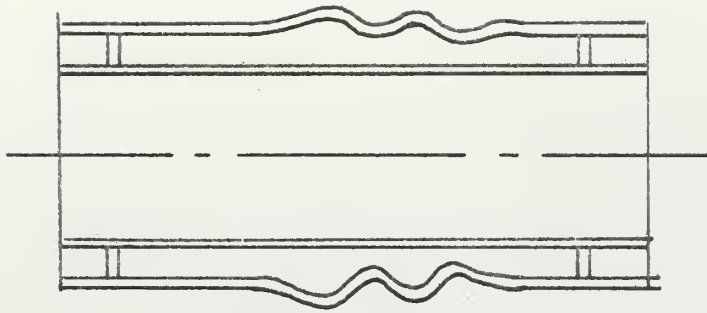
$$I_{\bar{c}} = \frac{bd^3}{12} + \frac{LT_o^3}{12} + \frac{LT_i^3}{12} + T_o L \left(\frac{d}{2} - \frac{T_o}{2} \right)^2 + T_i L \left(\frac{d}{2} - \frac{T_i}{2} \right)^2$$

$$A = (T_o L + T_i L + db)$$

$$y = (Y_{cg} - d/2)$$

$$Y_{cg} = \frac{T_o L \left(d - \frac{T_o}{2} \right) + db \left(\frac{d}{2} \right) + T_i L \left(\frac{T_i}{2} \right)}{(T_o L + db + T_i L)}$$

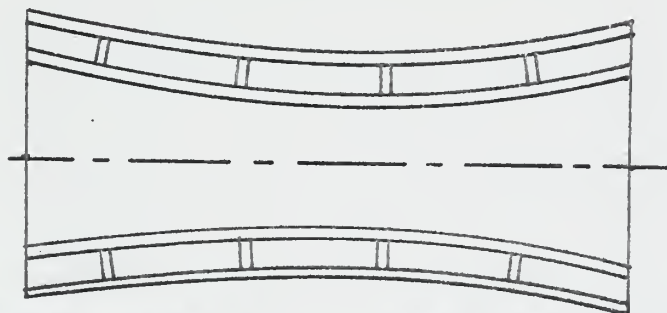
$$I_{eff} = I_{\bar{c}} - Ay^2$$



AXISYMMETRIC BUCKLING (BETWEEN WEBS)

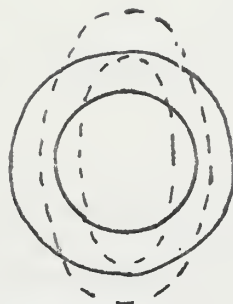


ASYMMETRIC (LOBAR) BUCKLING (BETWEEN WEBS)

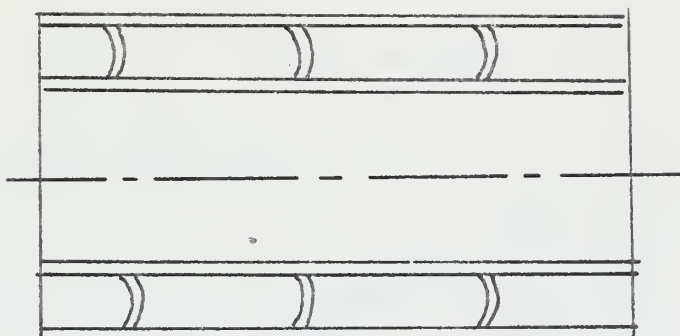


GENERAL INSTABILITY (CONCURRENT FAILURE
OF SHELLS AND WEBS)

FAILURE MODES (SHELL)



WEB (RING) BUCKLING



WEB (COMPRESSION) BUCKLING

FAILURE MODES (WEB)

PROCEDURE

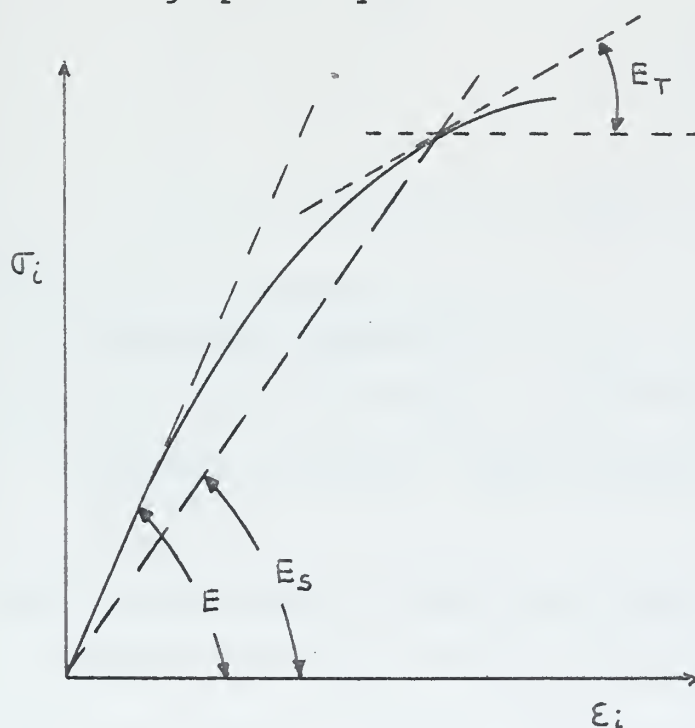
The buckling pressure is a function of the cylinder geometry and the secant and tangent moduli as determined from a stress-strain intensity diagram for the shell material.

Before the buckling equations can be used, E_s and E_T must be related to the applied pressure. The secant and tangent moduli are defined

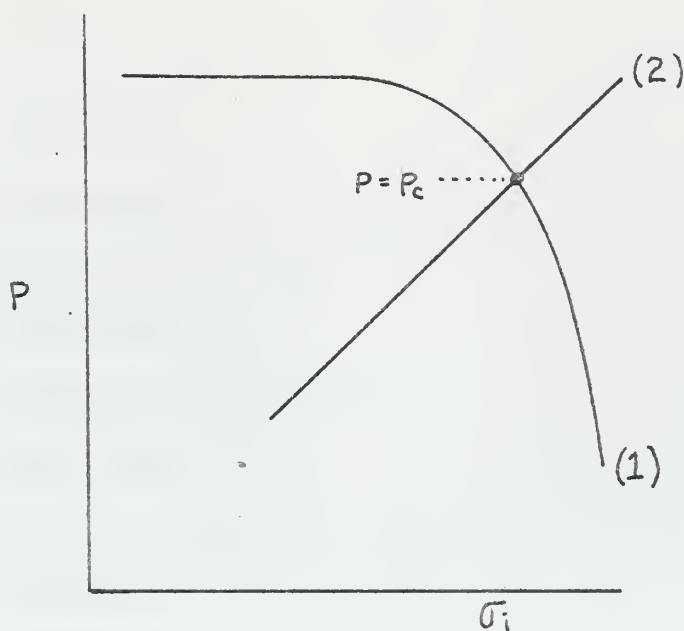
$$E_s = \sigma_i / \epsilon_i$$

$$E_T = d\sigma_i / d\epsilon_i$$

and are shown graphically:



Hence E_s and E_T are readily determined from the stress-strain curve (see Appendix C). Having determined E_s/E and E_T/E as functions of σ_1 , they must be applied to the hydrostatically loaded cylindrical shell. By expressing hydrostatic pressure in terms of the stress intensity, a relationship between E_s , E_T , and pressure will be established.



$$(1) P_c = f(\text{geometry}, E_s, E_T)$$

a buckling equation

$$(2) \sigma_1 = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2} = p \sqrt{K_1^2 + K_2^2 - K_1 K_2}$$

a stress intensity (Huber-Hencky-Von Mises) equation

The intersection of these two curves will predict failure by inelastic buckling if below the yield strength.

Thus the general solution technique is to (1) solve for the state of stress, and consequently a relationship

between σ_i and p , and then (b) solve the buckling modes as a function of $\sigma_i(E_S, E_T)$. The intersection of the curves being the critical pressure (see 1b).

1. Solution Technique.

a. *STRESS ANALYSIS*

INPUT DATA:

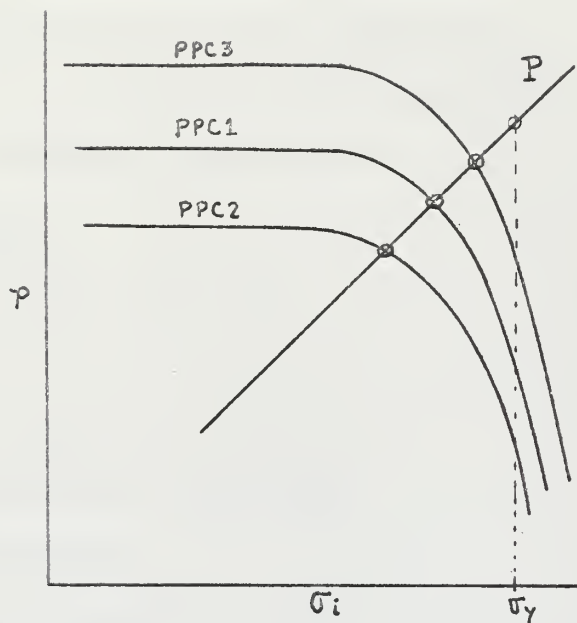
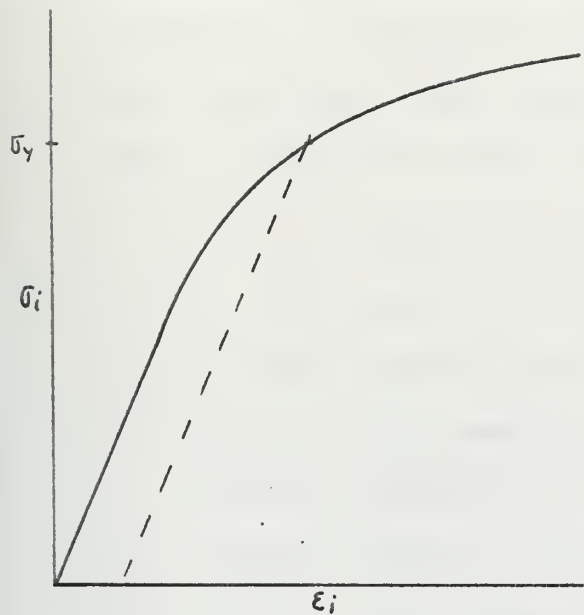
$R_O, R_I, T_O, T_I, b, L, \nu, p$

Let $p = 1.0$ psi

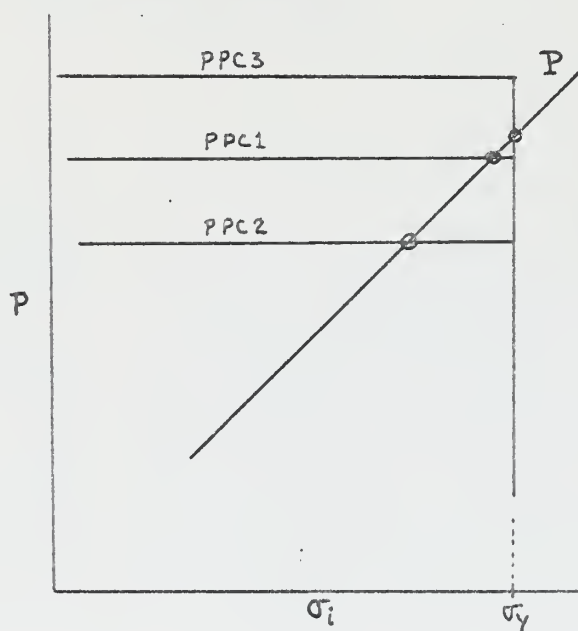
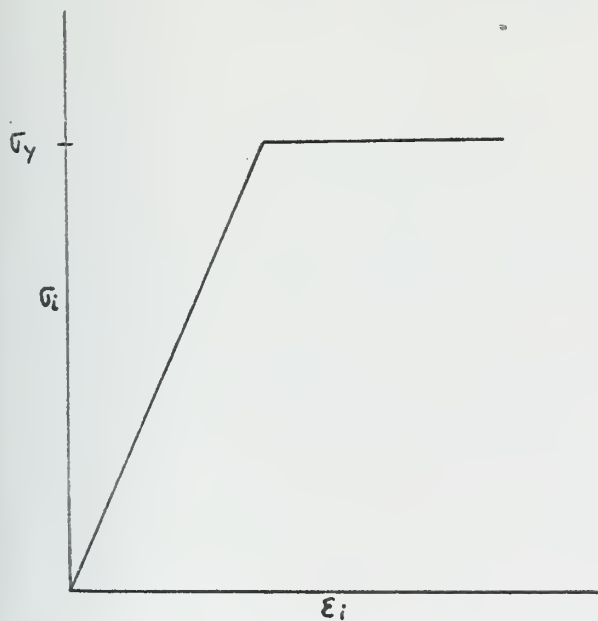
- (1) Formulate θ_O, θ_I
- (2) Formulate $F_{10}, F_{11}, F_{20}, F_{21} \dots$
- (3) Formulate g_O, g_i
- (4) Formulate R_w, d
- (5) Formulate M, N, P, Q
- (6) Formulate V_O, V_i
- (7) Solve simultaneous equations for H_O, H_i
- (8) Formulate HHO, HHI, RR
- (9) Formulate σ_{uo}, σ_{ui}
- (10) Formulate $B_{10}, B_{11}, B_{20}, B_{21} \dots$
- (11) Solve for the states of stress

b. FAILURE MODES

(1) INELASTIC MATERIALS



(2) ELASTIC - PERFECTLY PLASTIC MATERIAL



c. OPTIMUM PROPORTIONS

A simple optimization program was used to determine optimum proportions. As indicated in the flow diagram, a comparison is made between the design collapse pressure (PCO) and all the failure mode pressures (P_i). The failure mode pressures consist of:

$P \equiv$ yield at midbay

$PF \equiv$ yield at web-shell junction

$PWS \equiv$ yield in web

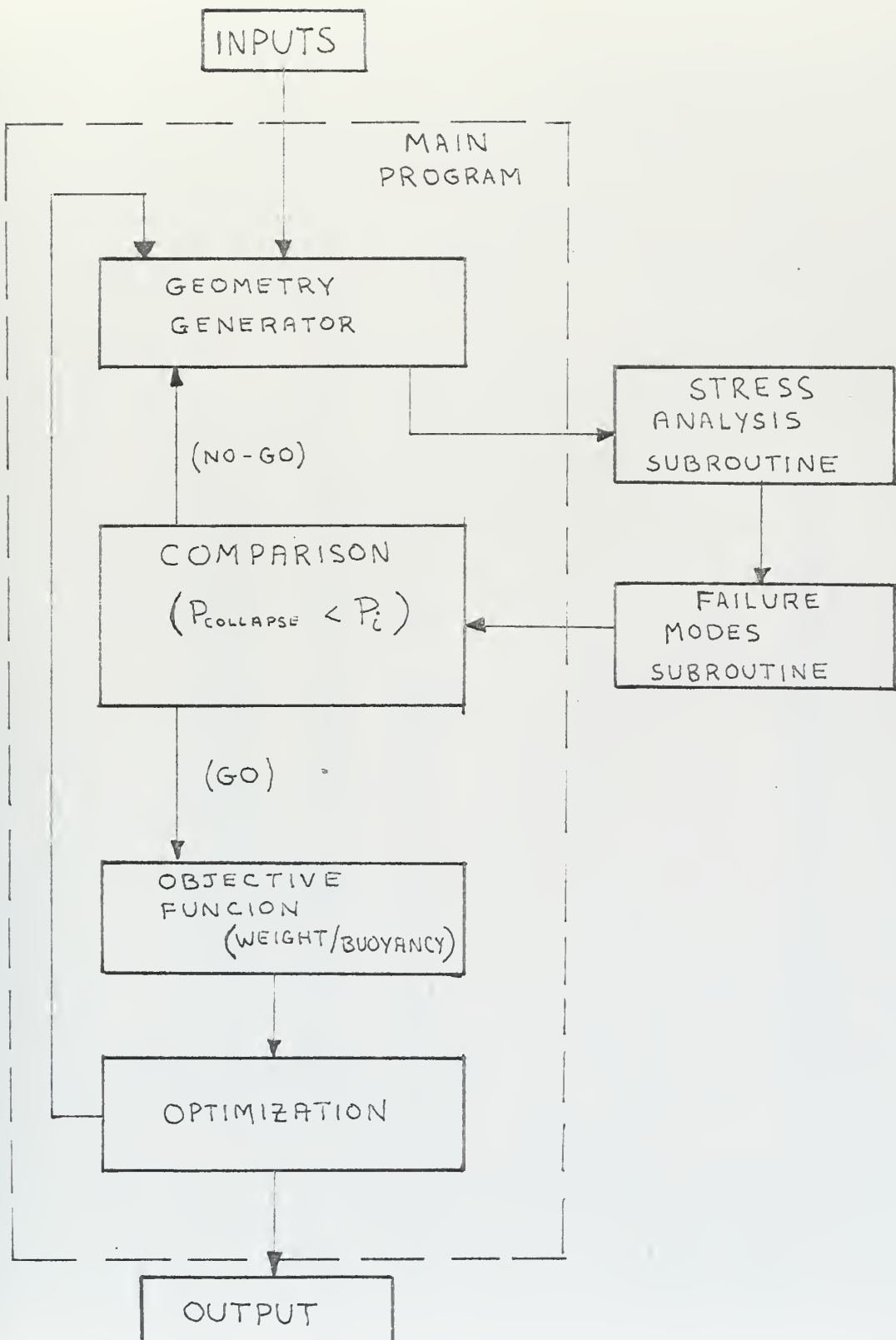
$PPC1 \equiv$ axisymmetric buckling, each shell

$PPC2 \equiv$ asymmetric buckling, each shell

$PPC3 \equiv$ general instability

$PPC4 \equiv$ web instability (ring)

$PPC5 \equiv$ web buckling (edge compression)




```

C      OPTIMIZATION PROGRAM FOR WEB-STIFFENED SANDWICH PRESSURE HULL
      DIMENSION RO(5),IO(5),II(5),B(5),WL(5)
      COMMON NUE,L,E,EZ,SIGMAY,LH
      REAL NUE,LB,IEFF,KR,KP
      61 READ(5,100) PCC
      100 FORMAT(F10.0)
      IF(PCC-1.0)3,3,60
      3 CALL EXIT
      60 READ(5,101) RHO,E,ET,EZ,SIGMAY,NUE,LB,RI,(RO(I),I=1,5)
      1,(TI(J),J=1,5),(TI(K),K=1,5),(B(L),L=1,5),(WL(M),M=1,5)
      101 FORMAT(F10.2,E10.2,2F10.2,F10.0,F10.0,2F10.2/5F10.2/
      1 5F10.2/5F10.2/5F10.2/5F10.2)
      WRITE(6,200) PCC,RHO,NUE,SIGMAY,RI
      200 FORMAT(' COLLAPSE PRESSURE=F10.0/5X,' DENSITIVE=F10.1/5X,' VUE=
      1 F10.2/5X,' SIGMAY=F10.0/5X,' INSIDE RADII S='F10.3)
      WRITE(6,250)
      250 FORMAT(5X,' RRO',4X,' ITU',4X,' TTI',4X,' HRW',4X,
      1 ' HTS',4X,' HS',4X,' BIS',4X,' PF',4X,' VJ')
      WB=10.00
      PI=3.1416
      DO 25 I=1,5
      DO 20 J=1,5
      DO 15 K=1,5
      DO 10 M=1,5
      DO 5 L=1,5
      RRO=R0(I)
      BB=B(L)
      TIU=TI(J)
      TTI=TI(K)
      WL=WL(M)
      RRI=RI
      CALL STAS(RRO,RRI,ITU,TTI,BB,WL,SX,SPHI,FX,FPHI,SSX,
      1 SSPHI,FFX,FFPHI,KR,KP,HRW,HTS,HS,BTS,IEFF,D,H,IS)
      RW=(RRO+RRI)/2.0
      DM=2.0*RW
      CALL FMUS(SX,SPHI,FX,FPHI,KR,KP,RRO,ITU,IEFF,D4,WL,

```



```

1  H,TS,BB,P,PF,PPC1,PPC2,PPC3,PPC4,PPC5,PWS,PND)
   IF(PCO-PF)4,4,7
4  IF(PCO-P)6,6,7
6  IF(PCO-PPC1)8,8,7
8  IF(PCO-PPC2)9,9,7
9  IF(PCO-PPC3)11,11,7
11 IF(PCO-PPC4)12,12,7
12 IF(PCO-PPC5)33,33,7
33 IF(PCO-PWS)34,34,7
7  GO TO 5
34 CALL FMCS(SSX,SSPH1,FFX,FFPH1,KR,KP,RR1,TTI,IEFT,D4,
   1  MWL,H,TS,RB,P,PF,PPC1,PPC2,PPC3,PPC4,PPC5,PWS,PND)
   IF(PCO-PF)2,2,14
2  IF(PCO-P)13,13,14
13 IF(PCO-PPC1)16,16,14
16 IF(PCO-PPC2)17,17,14
17 IF(PCO-PPC3)18,18,14
18 IF(PCO-PPC4)19,19,14
19 IF(PCO-PPC5)21,21,14
21 IF(PCO-PWS)22,22,14
14 GO TO 5
22 WD=2.0*RHC/64.4*(TTQ/RRC+(RRI/RRU)*(TTI/RRJ)+(RB/RRJ)
   1  *(D/(WWL+BB)))
   WRITE(6,40)RRC,TTQ,TTI,HRN,HTS,HS,BTS,PF,WD
400 FORMAT(2X,F8.3,2X,F8.3,2X,F8.3,2X,F8.3,2X,F8.3,2X,F8.
   1  3,2X,F8.3,2X,F8.3,2X,F8.3)
   IF(WD-WB)30,30,31
30 WB=WD
21 GO TO 5
5  CONTINUE
10 CONTINUE
15 CONTINUE
20 CONTINUE
25 CONTINUE
   WRITE(6,300)WB
300 FORMAT(' WEIGHT/BUOYANCY='F10.3/)

```


GC TJ 61
END


```

SUBROUTINE STAS(RC,RI,TC,TI,B,WL,MXCO,MPHIOU,FXUI,
1 FPHIOI,MXII,MPHII,FXIO,FPHIO,KR,KP,HRW,HTS,HS,BTS,
2 JEFF,D,H,TS)
COMMON NUC,E,ET,EZ,SIGMAY,LB
C STRESS ANALYSIS (STAS) OF WEB STIFFENED SANDWICH CYLINDRICAL SHELLS
REAL NUC,L,M,N,MXOU,MPHIO,MXJI,MPHIOI,MXI),MPHIU,
1 MXII,MPHII,IEFF,KR,KP
POP=1.0
L=WL
THEDAO=(3.0*(1.0-NUE**2.0))**0.25*L/(RO*TI)**0.50
THEDAI=(3.0*(1.0-NUE**2.0))**0.25*L/(RI*TI)**0.50
F10=THEDAO/2.0*(SINH(THEDAO)+SIN(THEDAO))/(COSH(THEDAO)-COS(THEDAO)
1 ))
F11=THEDAI/2.0*(SINH(THEDAI)+SIN(THEDAI))/(COSH(THEDAI)-COS(THEDAI)
1 ))
BEDAJ=THEDAO/2.0
BEDAI=THEDAI/2.0
F20=THEDAO*(COSH(BEDAO)*SIN(BEDAO)+SINH(BEDAO)*COS(BEDAO))/(
1 COSH(THEDAO)-COS(THEDAO))
F21=THEDAI*(COSH(BEDAI)*SIN(BEDAI)+SINH(BEDAI)*COS(BEDAI))/(
1 COSH(THEDAI)-COS(THEDAI))
F30=6.0*THEDAO/(3.0*(1.0-NUE**2.0))**0.5*(COSH(BEDAO)*SIN(THEDAO)-
1 SINH(BEDAO)*COS(BEDAO))/(COSH(THEDAO)-COS(THEDAO))
F31=6.0*THEDAI/(3.0*(1.0-NUE**2.0))**0.5*(COSH(BEDAI)*SIN(BEDAI)-
1 SINH(BEDAI)*COS(BEDAI))/(COSH(THEDAI)-COS(THEDAI))
F40=3.0*THEDAO/(3.0*(1.0-NUE**2.0))**0.5*(SINH(THEDAO)-SIN(THEDAO)
1)/(COSH(THEDAO)-COS(THEDAO))
F41=3.0*THEDAI/(3.0*(1.0-NUE**2.0))**0.5*(SINH(THEDAI)-SIN(THEDAI)
1)/(COSH(THEDAI)-COS(THEDAI))
GJ=-2.0*RO**2.0*F10/(TO*L)
GI=-2.0*RI**2.0*F11/(TI*L)
RW=(RO+RI)/2.0
D=(RJ-RI)+(TC+TI)/2.0
M=RW/B*(1.0+D/(2.0*RW))*(2.0*RW/D+D/(2.0*RW)-2.0*NUE)
N=RW/B*(1.0+D/(2.0*RW))*(2.0*RW/D+D/(2.0*RW)-2.0)
P=RW/B*(1.0+D/(2.0*RW))*(2.0*RW/D+D/(2.0*RW)+2.0)

```



```

Q=RW/B*(1.0-D/(2.0*RW))*(2.0*C*R4/D+D/(2.0*R4)+2.0*NUE)
VC=(RU/2.0)/(1.0+(FI/TU)*(RI/RJ))
VI=(FI/TU)*VC
HJ=((GI-Q)*(-(RQ**2.0/TU)*(1.0-(NUE/2.0)/(1.0+(FI/TU)*(RI/RJ)))+(
1 B*M/2.0)-(-N)*(NUE*RQ**2.0/(2.0*T1))*(1.0-(1.0/(1.0+(FI/TU)*(RI/
2 RU)))+B*p/2.0))/(GI-Q)*(GJ-M)-(-N)*(-P))
HI=(-P)*(-(RJ**2.0/TU)*(1.0-(NUE/2.0)/(1.0+(FI/TU)*(RI/RJ)))+(B*M
1 /2.0)-(GC-M)*(NUE*RQ**2.0/(2.0*T1))*(1.0-(1.0/(1.0+(FI/TU)
2 *(RI/RJ)))+(B*p/2.0))/((-N)*(-P)-(GI-Q)*(30-M))
SIGUJ=-PCP*RU/TU
SIGUI=-PCP*RI/II
BJC=2.0*F20-NUE*F30
BII=2.0*F21-NUE*F31
BJC=2.0*F20+NUE*F30
BII=2.0*F21+NUE*F31
BJC=F30
BII=F31
BJC=F40
BJC=2.0*F10+NUE*F40
BJC=2.0*F11-NUE*F41
BJC=F41
BJC=2.0*F11+NUE*F41
BJC=2.0*F11-NUE*F41
STRESS STATE ONE
MX00=(V0/R0+H0*B30/L)*SIG00-
MFI00=(1.0-H0*B10/L)*SIG00
FX01=(V0/R0+H0*B40/L)*SIG00
FPHI01=(1.0-H0*B60/L)*SIG00
STRESS STATE TWO
FX10=(V1/R1-H1*B41/L)*SIG1
FPHI10=-(-H1*B51/L)*SIG1
MX11=(V1/R1-H1*B31/L)*SIG1
MFI11=(-H1*B21/L)*SIG1
WER STRESS CALCULATION
FHC=H0+B/2.0
HHI=HI

```

C

C

C


```

DRI=RI-TI/2.C
DRO=RO+TO/2.O
RR=DRI/ORD
KR=((-HHI*RR**2.O+HHO)-(-HHI*RR+HHO*RR)**2)/(A*(1.O-RR**2.O))
KP=((-HHI*RR**2.O+HHO)+(-HHI*RR+HHO*RR)**2)/(B*(1.O-RR**2.O))
EFFECTIVE 1 CALCULATION
YCG=(TO*L*(O-TO/2.O)+D**2.O*B/2.O+TI**2.O*L/2.O)/(O*L+D*B+T*L)
A=YCG-O/2.O
G=C.O
      IF (A-G)20,20,21
20  LEFF=B*D**3.O/12.O+TO*L*(O/2.O-TO/2.O)**2.O+TI*L*(O/2.O-TI/2.O)
      1 **2.C
      GC TO 23
21  LEFF=B*D**3.O/12.O+TO*L*(O/2.O-TO/2.O)**2.O+TI*L*(O/2.O-TI/2.O)
      1 **2.O-(TO*L+D*B+TI*L)*(YCG-O/2.O)**2.O
23  TS=(TO+TI)/2.O
      S=L+B
      H=RO-RI
      HRW=H/RW
      RTS=H/TS
      HS=H/S
      RTS=3/TS
      RETURN
      END

```

C


```

SUBROUTINE FMCS(SX,SPHI,FX,FPHI,KR,KP,R,T,IEFF,DM,L,H,
1 TS,B,P,PF,PPC1,PPC2,PPC3,PPC4,PPC5,PWS,PND)
COMMON NUE,E,ET,EZ,SIGMAY,LB
REAL NU,NUE,L,LAMDA,IEFF,IGMAI,LB,KR,KP
PI=3.1416
P=SIGMAY/(SX*SX+SPHI*SPHI-SX*SPHI)**0.5
PF=SIGMAY/(FX*FX+FPHI*FPHI-FX*FPHI)**0.5
EZ=EZ**2.0/ET
FE=FX/SPHI
RW=DM/2.0
TEDA=1.23*(R*T)**0.5/L
NU=0.5-ES*(0.5-NUE)
PPC1=(ET*E**L**2.0/(12.0*(1.0-NU)**2.0)*SX*PI**2.0*R**2.0))
1 *(PI**4.0*R**2.0*1**2.0*(1.0+0.75*(ES/ET-1.0))/(**4.0+12.0)
2 *(1.0+(ES/ET-1.0))*(1.0-NU**2.0)/(1.0+0.75*(ES/ET-1.0)))*(-1.0)
PPC2=PI**2.0*E*ET*FE*ET/(3.0*SX*TEDA*(1.0-NU)**2.0)*K*(R*T)**0.5/
1 L)**2*(1.0+0.75*TEDA*(ES/ET-1.0)/(3.0-2.0*TEDA*(1.0-FE)))*(-1.0)
LAMDA=PI*R/LB
DO 7 K=1,5
PPC3=50000.0
N=K
VPC3(K)=(2.0*TS)*(ET*ES)**0.5*E/RW*(LAMDA**4.0/((N**2.0
1 -1.0+LAMDA**2.0/2.0)*(N**2.0+LAMDA**2.0))*(1.0-0.75*(
2 1.0-ES/ET)*LAMDA**4.0/(N**2.0+LAMDA**2.0)**2.0))+
3 (N**2.0-1.0)*ET*E*IEFF/(RW**3.0*L)
C3=VPC3(K)
IF(C3-PPC3)3,3,7
3 PPC3=C3
7 CONTINUE
PPC4=24.0*E*IEFF/(DM**3.0*L*(1.0-NUE**2.0))*ET
PPC5=(4.0*(PI**2.9)*E*ET**B**2.0)/(12.0*(1.0-NU**2.0)*(KP+4.0*KK)
1 *H**2.0)
PWS=SIGMAY/(KR*KR+KP*KP-KR*KP)**0.5
PND=4.0*TS*SIGMAY/DM
RETURN
END

```



```

C STRESS ANALYSIS (STAS) OF WEB STIFFENED SANDWICH
C CYLINDRICAL SHELLS
REAL NUC,L,M,N,MXUU,MPHICU,MXOI,MPHIOI,MXIO,MPHIO,
1 MXII,MPHIII,IEFF,KR,KP
DIMENSION RRC(10),RRI(10),TTO(10),TTI(10),BB(10),WL
1 (10),PNUE(10)
READ(5,700) RHC
700 FORMAT(F10.2)
READ(5,100) (RRO(I),RRI(I),TTO(I),TTI(I),BB(I),WL(I),
1 PNUE(I),I=1,9)
100 FORMAT(7F10.2/7F10.2/7F10.2/7F10.2/7F10.2/
1 7F10.2/7F10.2/7F10.2)
DO 10 I=1,9
RQ=RRQ(I)
RI=RRI(I)
IC=TTO(I)
TI=TTI(I)
B=BB(I)
L=WL(I)
NUE=PNUE(I)
WRITE(6,200) RQ,RI,TD,TI,B,L
200 FORMAT(' STRESS ANALYSIS OF A WEB STIFFENED SANDWICH
1 CYLINDRICAL PRESSURE HULL','/' OUTSIDE RADIUS(IN.)=',
2 F10.2/' INSIDE RADIUS(IN)='F10.2/' OUTSIDE THICKNESS
3 (IN)='F10.2/' INSIDE THICKNESS='F10.2/' WEB
4 THICKNESS(IN)='F10.2/' WEB SPACING(IN)='F10.2//2X,
5 ' MXUU',2X,' MPHIOI',2X,' MXOI',2X,' MPHIOI',2X,
6 ' MXCI',2X,' MPHIII',2X,' MXII',2X,' MPHIII'/'
POP=1.0
THEDAQ=(3.0*(1.0-NUE**2.0)**0.25*L/(RQ*TI)**0.50
THEDAI=(3.0*(1.0-NUE**2.0)**0.25*L/(RI*TI)**0.50
F1J=THEDAQ/2.0*(SINH(THEDAQ)+SIN(THEDAJ))/(COSH(THEDAQ)
1 -COS(THEDAJ))
F1I=THEDAI/2.0*(SINH(THEDAI)+SIN(THEDAI))/(COSH(THEDAI)
1 -COS(THEDAI))
BEDAJ=THEDAQ/2.0

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BEDAI=THEDAI/2.0
F2U=THEDAC*(COSH(BEDAU)*SIN(BEDAU)+SINH(BEDAU)*COS
1(BEDAU))/(COSH(THEDAU)-COS(THEDAU))
F2I=THEDAI*(COSH(BEDAI)*SIN(BEDAI)+SINH(BEDAI)*COS
1(BEDAI))/(COSH(THEDAI)-COS(THEDAI))
F3U=6.0*THEDAU/(3.0*(1.0-NUE**2.0))*0.5*(COSH(B+DAU))
1*SIN(BEDAU)-
2 SINH(BEDAU)*COS(BEDAU)/(COSH(THEDAU)-COS(THEDAU))
F3I=6.0*THEDAI/(3.0*(1.0-NUE**2.0))*0.5*(COSH(BEDAI)
1*SIN(BEDAI)-
2 SINH(BEDAI)*COS(BEDAI))/(COSH(THEDAI)-COS(THEDAI))
F4U=3.0*THEDAU/(3.0*(1.0-NUE**2.0))*0.5*(SINH(THEDAU)
1-SIN(THEDAU))/(COSH(THEDAU)-COS(THEDAU))
F4I=3.0*THEDAI/(3.0*(1.0-NUE**2.0))*0.5*(SINH(THEDAI)
1-SIN(THEDAI))/(COSH(THEDAI)-COS(THEDAI))
GJ=-2.0*RU**2.0*FI0/(TD*L)
GI=-2.0*RI**2.0*FI1/(TI*L)
RN=(RO+RI)/2.0
O=(RO-RI)+(IC+TI)/2.0
M=RW/B*(1.0+O/(2.0*RW))*((2.0*RW/D+D)/(2.0*RW)-2.0*NUE)
N=RW/B*(1.0+P/(2.0*RW))*((2.0*RW/D+D)/(2.0*RW)-2.0)
P=RW/B*(1.0-E/(2.0*RW))*((2.0*RW/D+D)/(2.0*RW)+2.0)
Q=RW/B*(1.0-D/(2.0*RW))*((2.0*RW/D+D)/(2.0*RW)+2.0*NUE)
VU=(RU/2.0)/(1.0+(TI/TO))*((RI/RO)
VI=(TI/TO)*VE
HJ=((GI-Q)*(-(RO**2.0/TO)*(1.0-(NUE/2.0)/(1.0+(TI/TO)
1*(RI/RO)))+B**M/2.0)-(-N))*((NUE*KJ**2.0/(2.0*TI))*
2(1.0-(1.0/(1.0+(TI/TO)*(RI/RO)))+B**P/2.0))/(GI-
3 GO-M)-(-N))*(-P))
HI=(((-P)*(-(PJ**2.0/TO)*(1.0-(NUE/2.0)/(1.0+(TI/TO)*
1(FI/RO)))+B**M/2.0)-(GO-M))*((NUE*RO**2.0/(2.0*TI))*
2(1.0-(1.0/(1.0+(TI/TO)*(RI/RO)))+B**P/2.0))/((-N)*
3(-P)-(GI-Q)*(GO-M))
SIGUD=-POP*RC/TC
SIGUI=-POP*RI/TI
3IU=2.0*F2U-NUE*F30

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B1I=2.0*F2I-NUE*F3I
B2C=2.0*F2C+NUE*F30
B2I=2.0*F2I+NUE*F3I
B30=F30
B3I=F3I
B40=F40
B50=2.0*F1C+NUE*F40
B60=2.0*F1C-NUE*F40
B4I=F4I
R5I=2.0*F1I+NUE*F4I
B6I=2.0*F1I-NUE*F4I
STRESS STATE ONE
FX00=(V0/R0-H0*B40/L)*SIGU
FPH10=(1.0-H0*B50/L)*SIGU
MX00=(V0/R0+H0*B30/L)*SIGU
MPH10=(1.0-H0*B10/L)*SIGU
STRESS STATE TWO
FX0I=(V0/R0+H0*B40/L)*SIGU
FPH1I=(1.0-H0*B60/L)*SIGU
MX0I=(V0/R0+H0*B30/L)*SIGU
MPH1I=(1.0-H0*B20/L)*SIGU
STRESS STATE THREE
FX10=(V1/R1-F1*B41/L)*SIGU
FPH1U=(-H1*B51/L)*SIGU
MX10=(V1/R1+H1*B31/L)*SIGU
MPH1U=(-H1*B11/L)*SIGU
STRESS STATE FOUR
FX1I=(V1/R1+H1*B41/L)*SIGU
FPH1I=(-H1*B61/L)*SIGU
MX1I=(V1/R1-F1*B31/L)*SIGU
MPH1I=(-H1*B21/L)*SIGU
WEB STRESS CALCULATION
FHC=HL+R/2.0
PHI=HI
DRI=KI-T1/2.0
CR1=R0+TC/2.0

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RR=DRI/CRG
KR=(((-HHI*RR**2.0+HHQ)-(-HHI*RR+HHQ*RR)*RR)/(B*(1.0-RR
1 **2.0))
KP=(((-HHI*RR**2.0+HHQ))+(-HHI*RR+HHQ*RR)*RR)/(B*(1.0-RR
1 **2.0))
C
EFFECTIVE I CALCULATION
YCG=(TO*L*(D-TU/2.0)+D**2.0*B/2.0+TI**2.0*L/2.0)/(TO*L
1 +D*B+TI*L)
A=YCG-D/2.0
G=0.0
IF(A-G)20,20,21
20 IEFF=B*D**3.0/12.0+TU*L*(D/2.0-TU/2.0)**2.0+TI*L*(D/
1 2.0-TI/2.0)**2.0
GO TO 23
21 IEFF=B*D**3.0/12.0+TO*L*(D/2.0-TU/2.0)**2.0+TI*L*(D/
1 2.0-TI/2.0)**2.0-(TO*L+TI*L)*(YCG-D/2.0)**2.0
23 WRITE(6,300) AXCI,APHIO,MAXI,MPHIO,MXIO,MPHIO,MXII,
1 MPHII
300 FJRMAT(1X,F7.2,1X,F7.2,1X,F7.2,1X,F7.2,1X,F7.2,1X,
1 F7.2,1X,F7.2,1X,F7.2)
WRITE(6,400)
400 FJRMAT(2X,'FXJ',2X,'FPHIO',2X,'FXI',2X,'FPHIO',
1,2X,'FXIC',2X,'FPHIO',2X,'FXII',2X,'FPHII')
WRITE(6,300) FXCI,FPHIO,FXCI,FPHIO,FXCI,FPHIO,FXII,
1 FPHII
IS=(TU+TI)/2.0
S=L+B
H=RO-RI
PRW=H/RW
PTS=H/IS
PS=H/S
BTS=B/IS
WB=2.0*RHO/64.4*(TO/RO+(RI/RJ)*(TI/RO)+(RN/RJ)*(3/RC)*
1 (D/S))
WRITE(6,500)
500 FJRMAT(5X,'PRW',5X,'HS',5X,'HTS',5X,'BTS',5X,

```



```
1  I EFF, 5X, ' D' / )  
   WRITE(6,600)HRW,HS,HTS,BTS,IEFF,D  
600 FORMAT(2X,4F10.3,F10.1,F10.3)  
   WRITE(6,800) WB  
800 FORMAT(5X, ' WB='F10.3 / )  
   WRITE(6,900)KR,KP  
900 FORMAT(5X, ' KR='F10.3,5X, ' KP='F10.3 / )  
10 CONTINUE  
   CALL EXIT  
   END
```



```

C  FAILURE MODES CRITERIA(FMCS)  FOUR STATES OF STRESS
  REAL NU,NUE,L,LAMDA,IEFF,IGMAI,LB,KR,KP
  DIMENSION IGMAI(10),EET(10),EES(10),SX(2),SPHI(2),
    1 FX(2),FPHI(2),R(2),T(2),VPC3(5)
  READ(5,102) (IGMAI(J),EET(J),EES(J),J=1,10)
102 FORMAT(3F10.2/3F10.2/3F10.2/3F10.2/3F10.2/3F10.2/3F10.2/3F10.2)
  12/3F10.2/3F10.2/3F10.2)
  10 READ(5,100) (SX(I),SPHI(I),FX(I),FPHI(I),R(I),T(I),
    1 I=1,2)
100 FORMAT(2F10.2/2F10.2/2F10.2/2F10.2/2F10.2/2F10.2)
  IF(SX(1)-1.0)8,8,9
  8 CALL EXIT
  9 READ(5,101) IEFF,D,L,NUE,LB,SIGMAY,E,KR,KP,B
101 FORMAT(5F10.2,F10.0,E10.2/3F10.2)
  WRITE(6,103) IEFF,D,L,NUE,LB,SIGMAY,E,B
103 FORMAT(5F10.2,F10.0,E10.2,F10.2)
  WRITE(6,200)
200 FORMAT(20X,' GRAPHICAL DATA FOR FAILURE MODES',//7X,
  1 ' P',7X,' PF',7X,' PPC1',6X,' PPC2',6X,' PPC3',6X,
  2 ' PPC4',6X,' PPC5',6X,' SIGMAI'//)
  DM=R(1)+R(2)
  RW=DM/2.0
  H=R(1)-R(2)
  PI=3.1417
  DO 5 I=1,2
  WRITE(6,400)I
400 FORMAT(' STATE OF STRESS='I2/)
  DO 6 J=1,10
  SIGMAI=IGMAI(J)*SIGMAY
  ET=EET(J)
  ES=EES(J)**2.0/ET
  P=SIGMAI/(SX(I)*SX(I)+SPHI(I)*SPHI(I)-SX(I)*SPHI(I))
  1 **0.5
  PF=SIGMAI/(FX(I)*FX(I)+FPHI(I)*FPHI(I)-FX(I)*FPHI(I))
  1 **0.5
  NU=0.5-ES*(0.5-NUE)

```



```

PPC1=(ET*E*L**2.0/(12.0*(1.0-NU**2.0)*SX(I)*PI**2.0*R
1 (I)**2.0))
2 *(PI**4.0*R(I)**2.0*T(I)**2.0*(1.0+.75*(ES/ET-1.0)))/
3 L**4.0+12.0
4 *(1.0+(ES/ET-1.0))*(1.0-NU**2.0)/(1.0+.75*(ES/ET-1.0)))
FE=SX(I)/SPHI(I)
TEDA=1.23*(R(I)*T(I)**.5/L
PPC2=PI**2.0*E*ET*FE*T(I)/(3.0*SX(I)*TEDA*(1.0-NU**2.0)
1 *R(I))*
2 ((R(I)*T(I)**0.5/L)**2.0*(1.0+.75*TEDA*(ES/ET-1.0)))/
3 (3.0-2.0*TEDA*(1.0-FE))
LAMDA=PI*R(I)/LB
DO 7 K=1,5
PPC3=50000.0
N=K
VPC3(K)=(T(1)+T(2))*(ET*ES)**0.5*E/RW*(LAMDA**4.0/(
1 (N**2.0-1.0+
2 LAMDA**2.0/2.0)*(N**2.0+LAMDA**2.0)*(1.0-.75*(1.0-
3 ES/ET)*LAMDA**
4 4.0/(N**2.0+LAMDA**2.0)**2.0)))+(N**2.0-1.0)*ET*E*
5 IEFF/(RW**3.0*L)
C3=VPC3(K)
IF(C3-PPC3)3,3,7
3 PPC3=C3
7 CONTINUE
PPC4=24.0*E*IEFF/(DM**3.0*L*(1.0-NU**2.0))*ET
PPC5=(4.0*(PI**2.0)*E*ET*B**2.0)/(12.0*(1.0-NU**2.0))*
1 (KP+4.0*KR)*H**2.0)
PND=2.0*(T(1)+T(2))*SIGMAY/DM
PWS=SIGMAY/(KR*KR+KP*KP-KR*KP)**0.5
WRITE(6,500)P,PF,PPC1,PPC2,PPC3,PPC4,PPC5,SIGMAI
500 FORMAT(2X,F10.1,1X,F10.1,1X,F10.1,1X,F10.1,1X,F10.1,1X,
1 F10.1,1X,F10.1,1X,F10.2/)
6 CONTINUE
5 CONTINUE
WRITE(6,501)PWS,PND

```

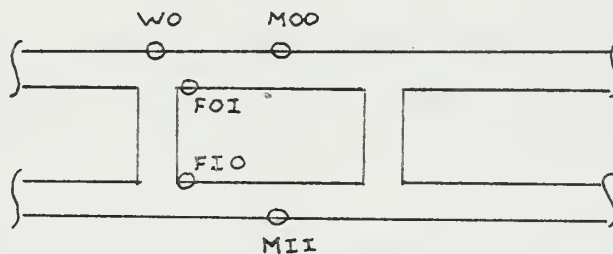


```
501 FORMAT(5X,' PWS='F10.1/5X,' PND='F10.1/)  
GO TO 10  
END
```


RESULTS

1. Geometric similitude was determined to exist. The stress analysis and failure mode programs output are in Appendix D for three geometrically similar structures. Graph 1 shows the failure modes for these three geometrically similar structures made from HY-160 steel. The effect of the inelastic range of this material on the buckling modes is indicated. The example has underdeveloped webs (PWS, PPC4).

2. Maximum stress states were found to be located as shown below.



3. The results are plotted in graphical form. The results are divided basically to show (a) the failure modes of the shells, (b) the failure modes of the webs, and (c) optimum proportions and weight/displacement results. The presentation of the results in this form was (1) to facilitate comparison with ring-stiffened cylinders, and (2) to

show the relationship of the various failure modes.

The form of the results are very similar to those of ring-stiffened cylinders.

4. Graphs 2 through 7 are sample results of the stress and buckling analysis for the shell portion of the web-stiffened sandwich cylinder using HY-80 steel.

5. Graphs 8 through 11 are sample results for the web portion non-dimensionalized by P_{ND}

where

$$P_{ND} = \frac{2(T_O + T_i) \sigma_y}{D_M}$$

The material was HY-80 steel.

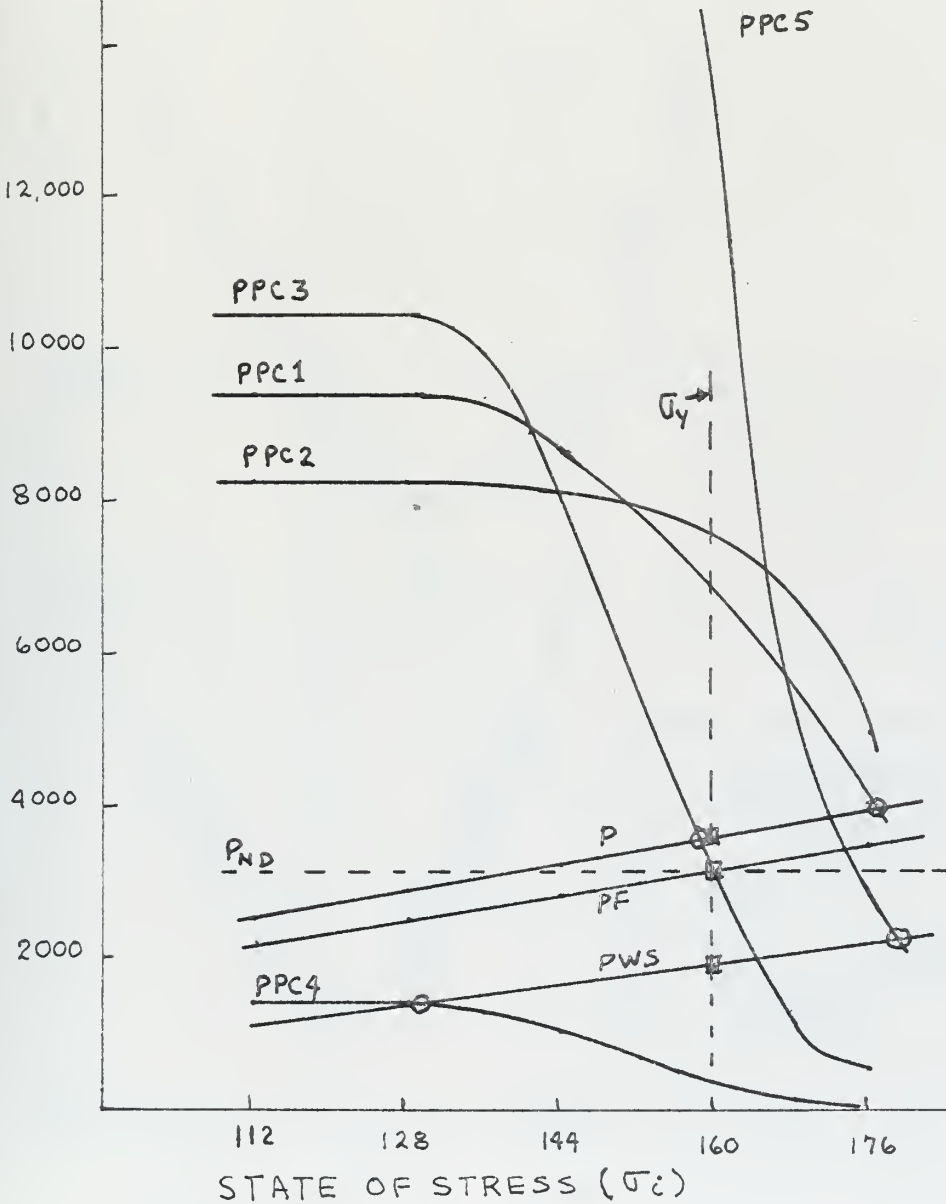
6. Graphs 12 and 13 show the effect of yield strength and the modulus of elasticity on shell failure modes respectively.

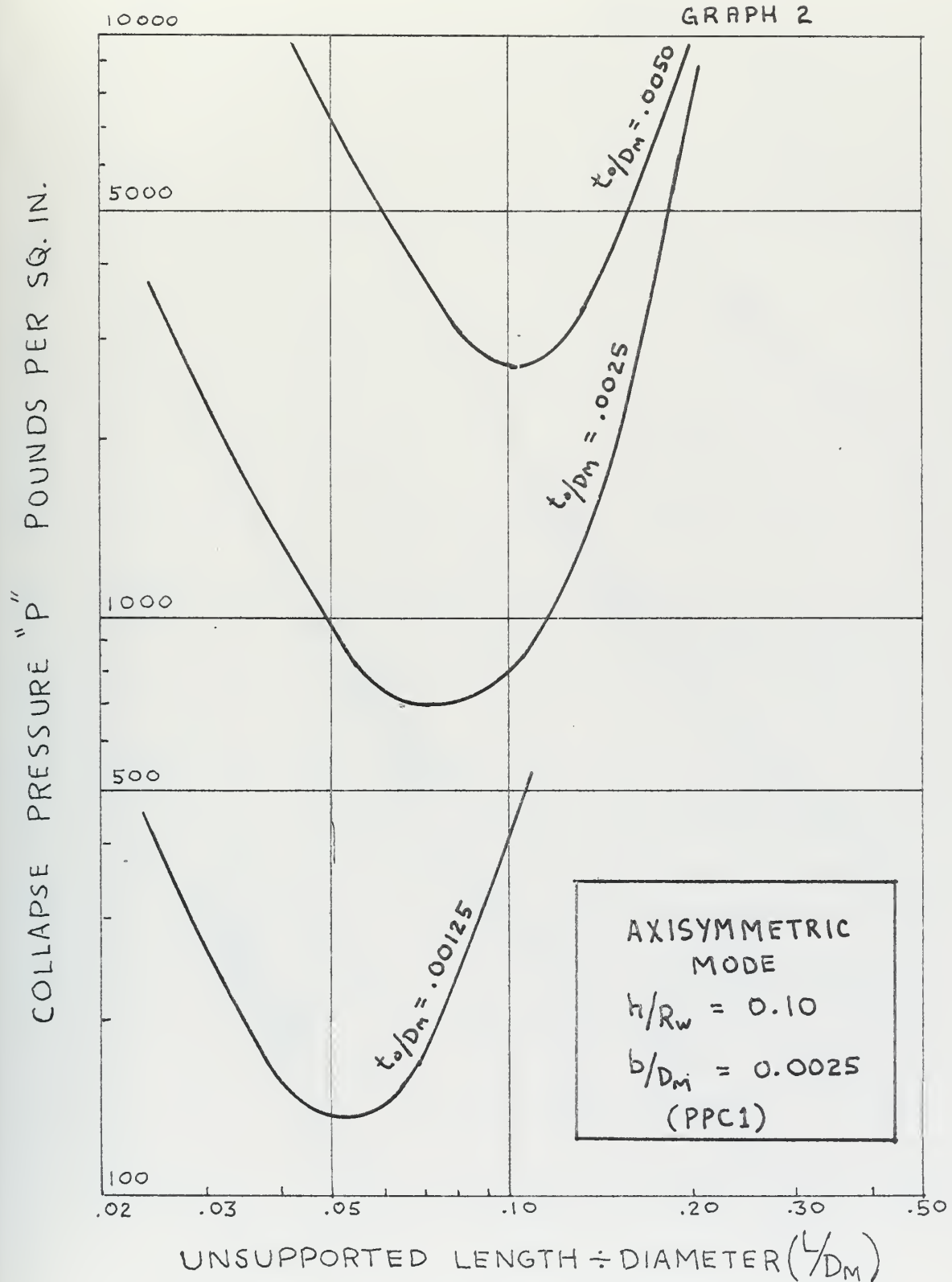
7. Graphs 14 and 15 and Tables 1 and 2 are concerned with the optimization results. The optimum proportions for ring-stiffened cylinders shown in Graph 15 were based upon using 92, 92A criteria for shell yield (outer fibers reaching yield stress).

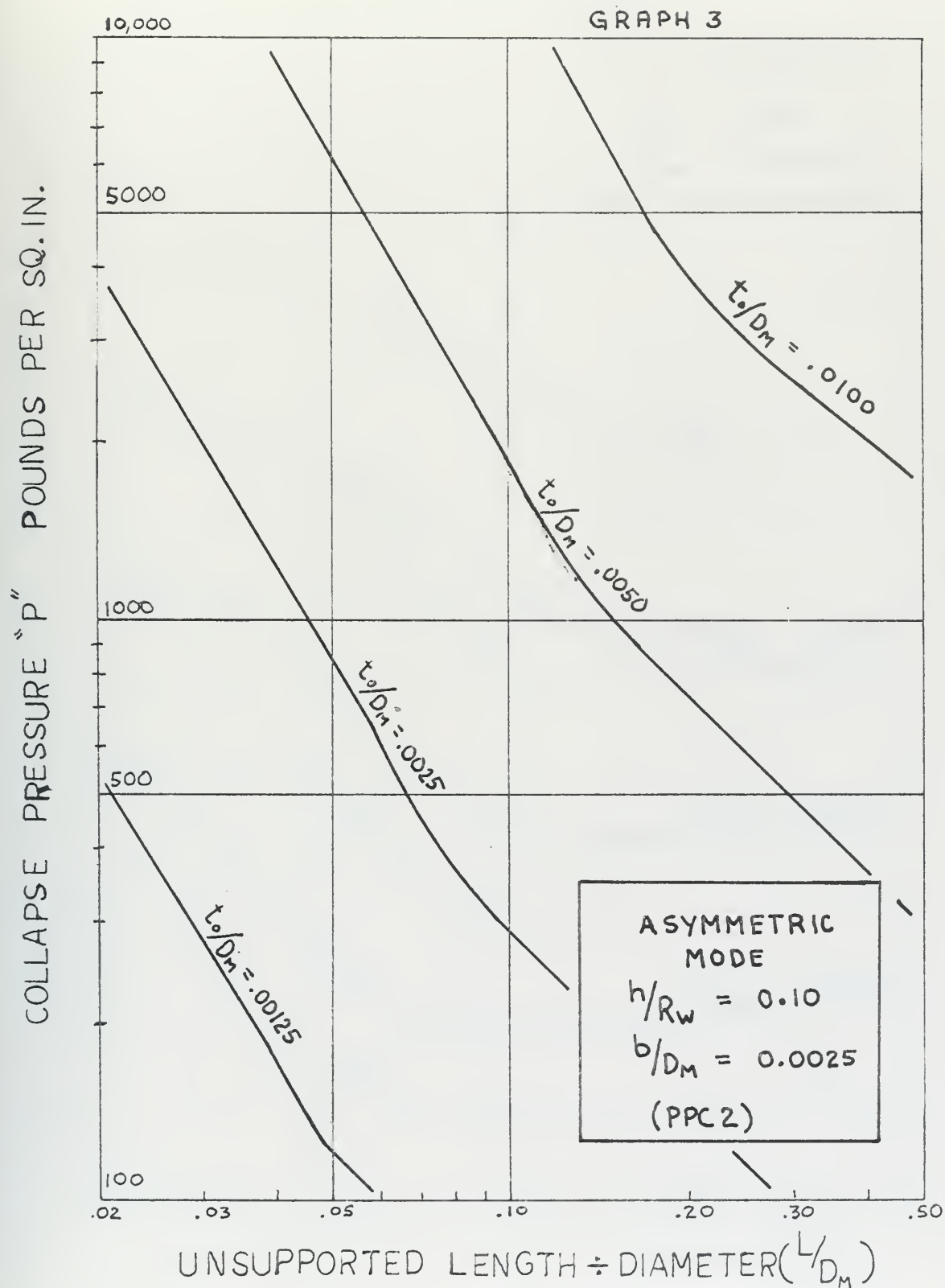
8. For a given hydrostatic loading and hull diameter the shell thicknesses for the web-stiffened sandwich are approximately half that of a ring-stiffened cylinder.

COLLAPSE PRESSURE "P" POUNDS PER SQ. IN.

FAILURE MODES
INELASTIC ANALYSIS
HY-160 STEEL







10000

GRAPH 4

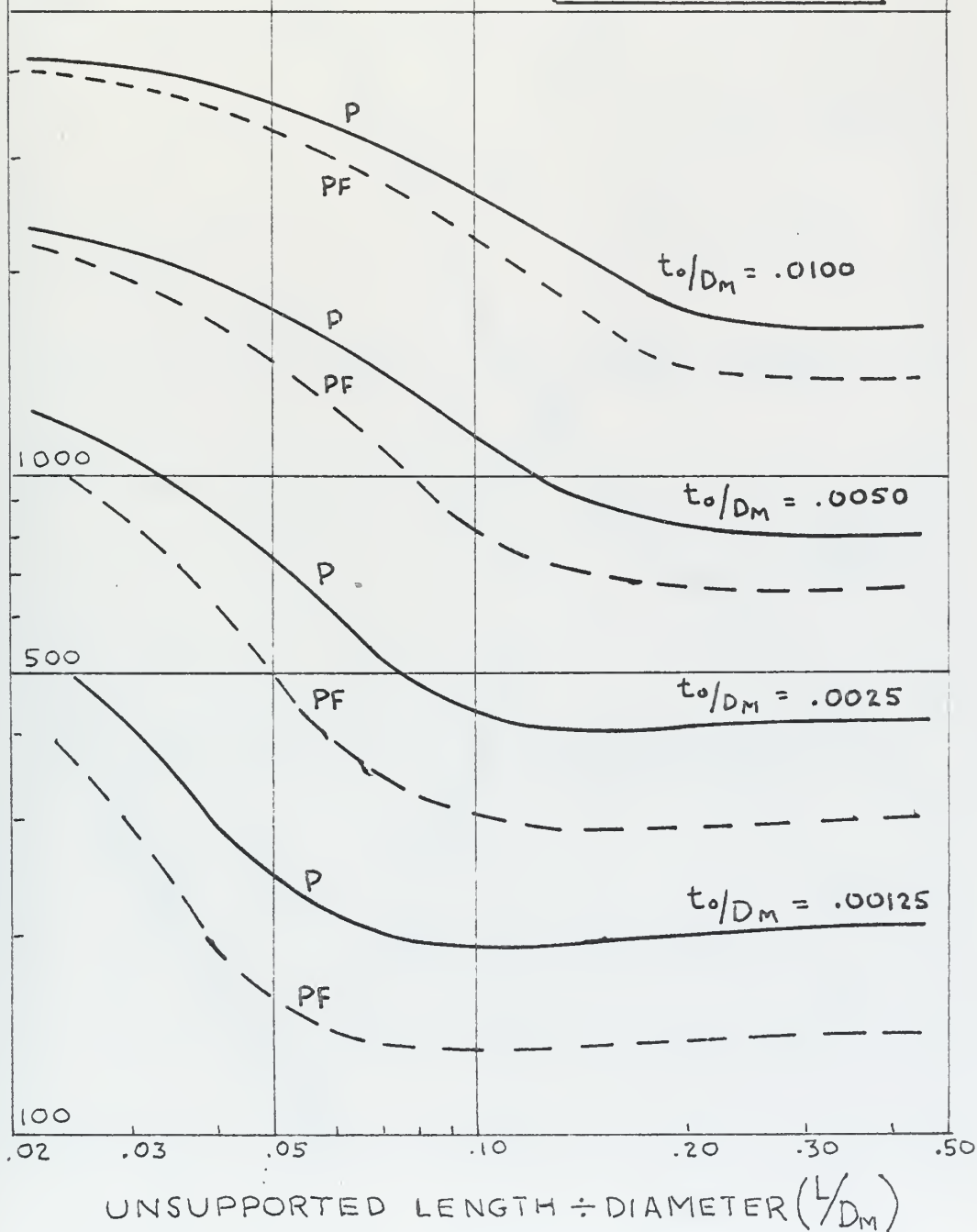
SHELL YIELD

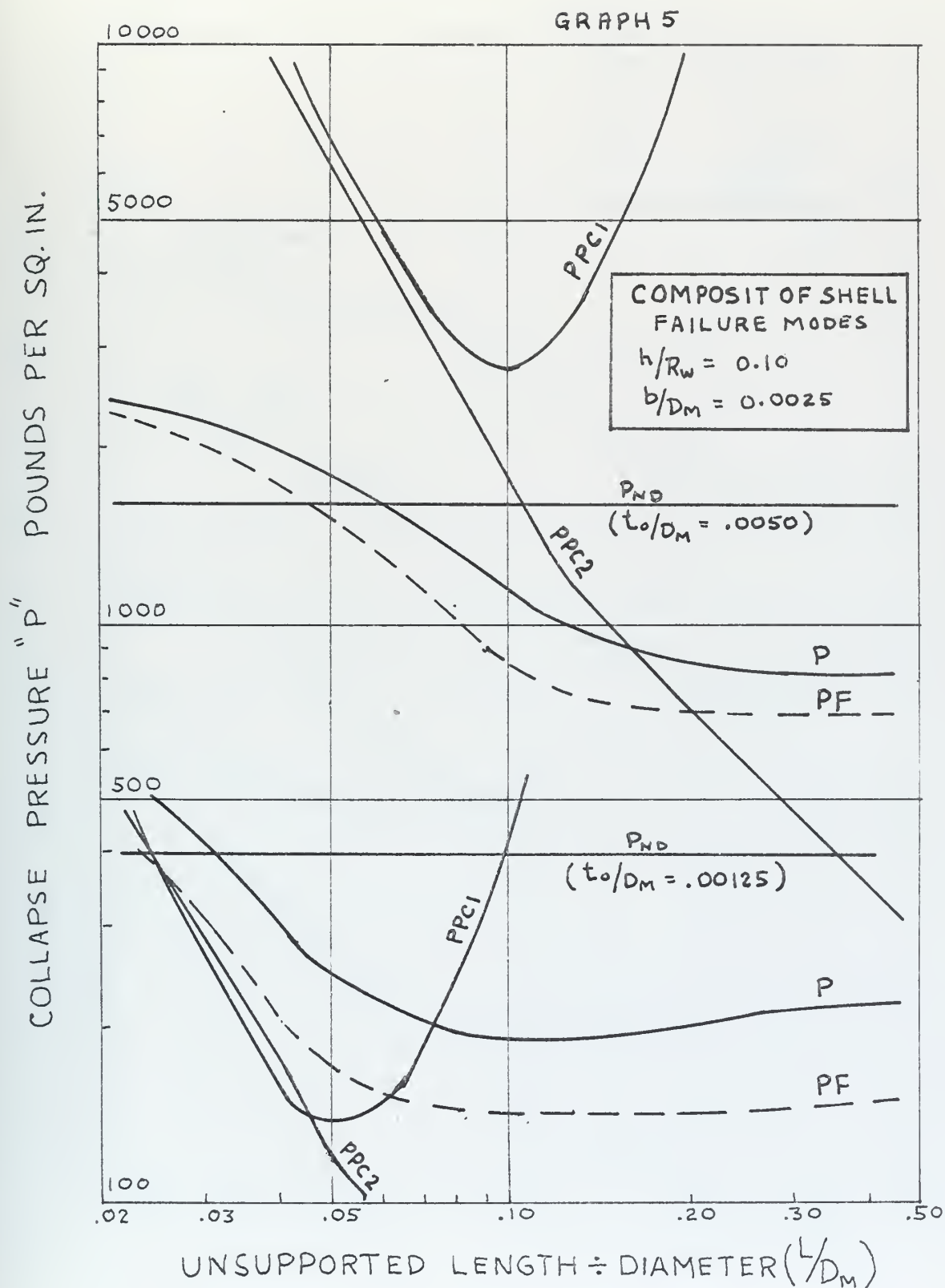
$$h/R_w = 0.10$$

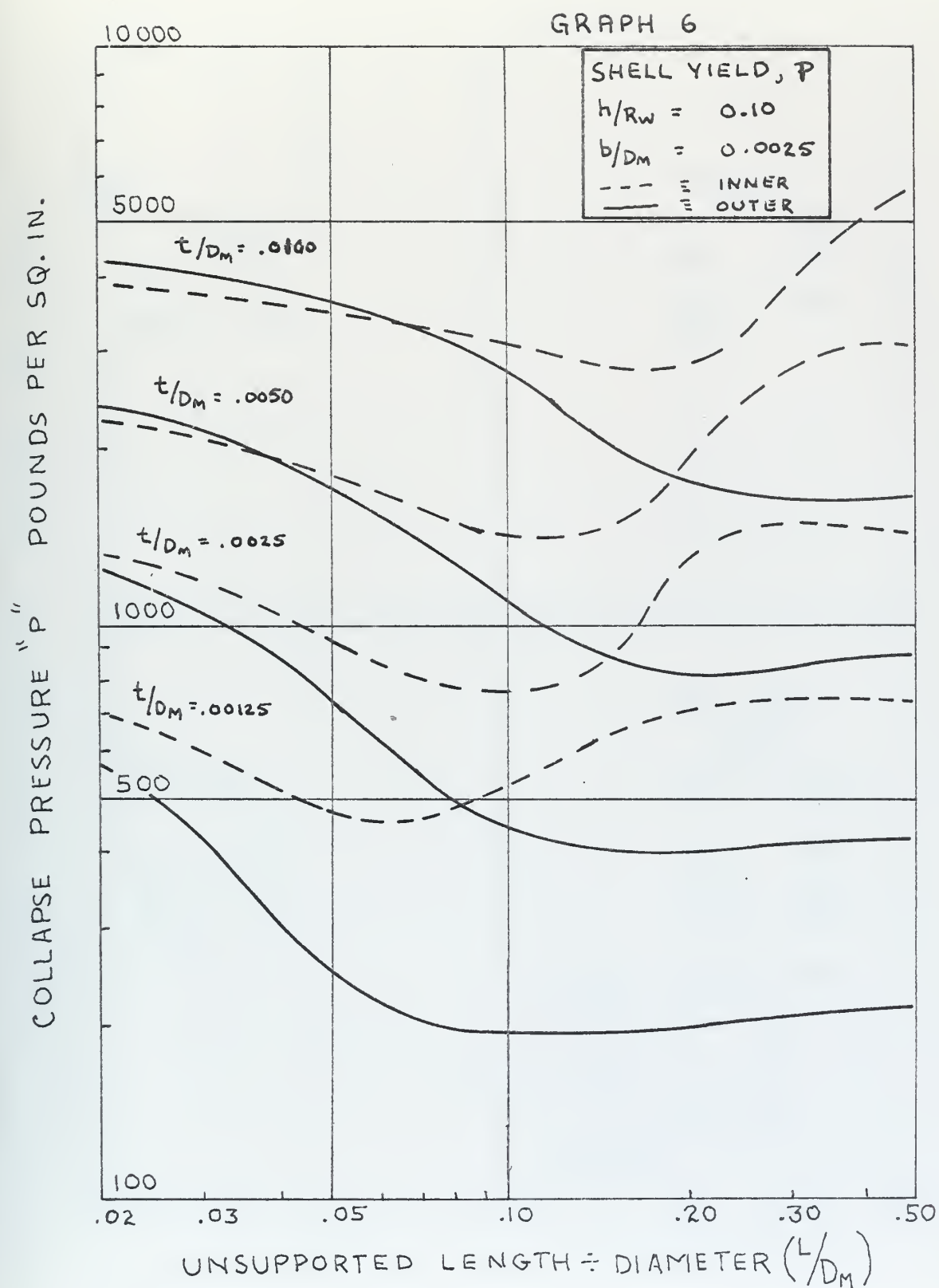
$$b/D_m = 0.0025$$

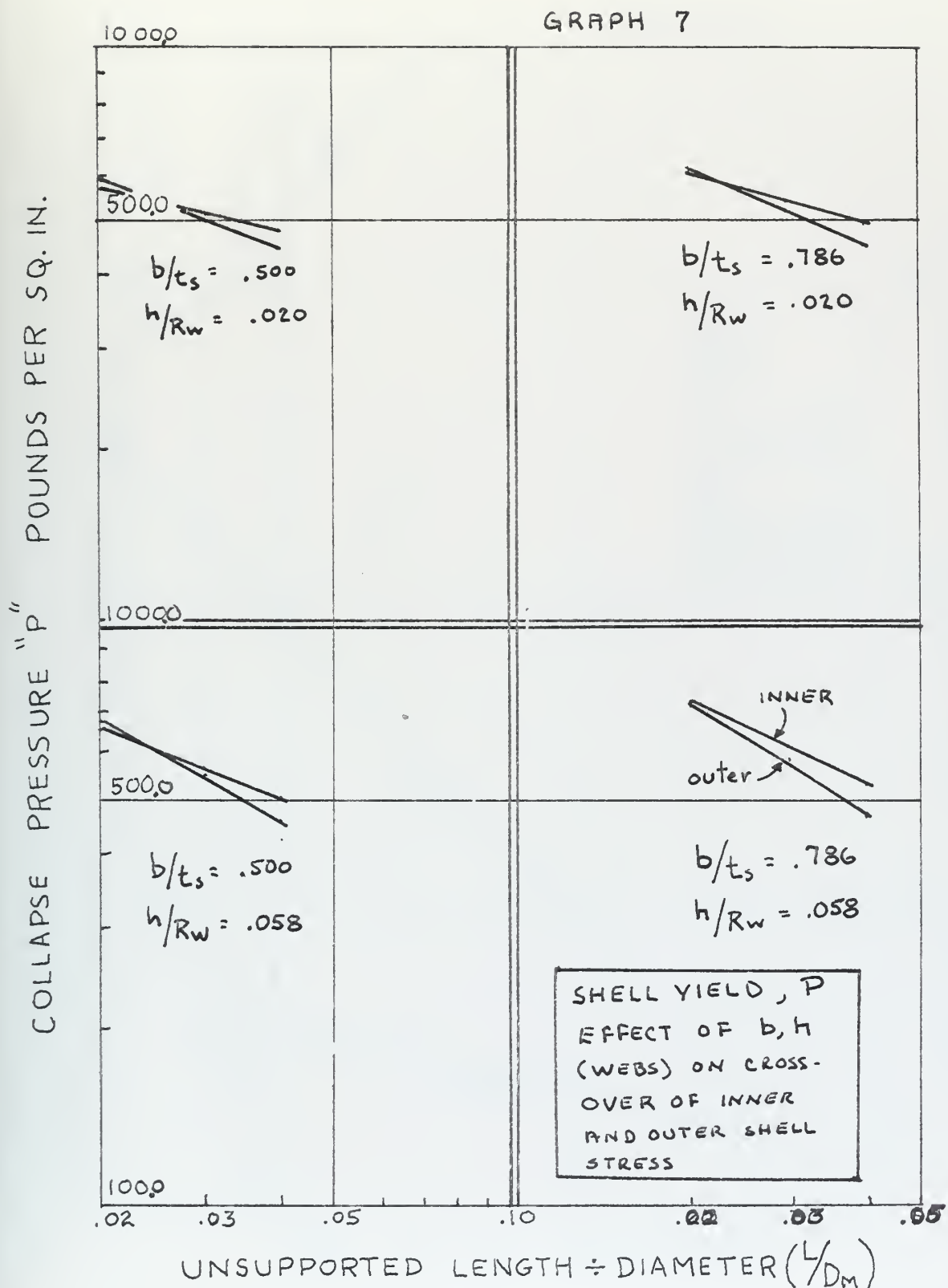
5000

COLLAPSE PRESSURE "P" POUNDS PER SQ. IN.

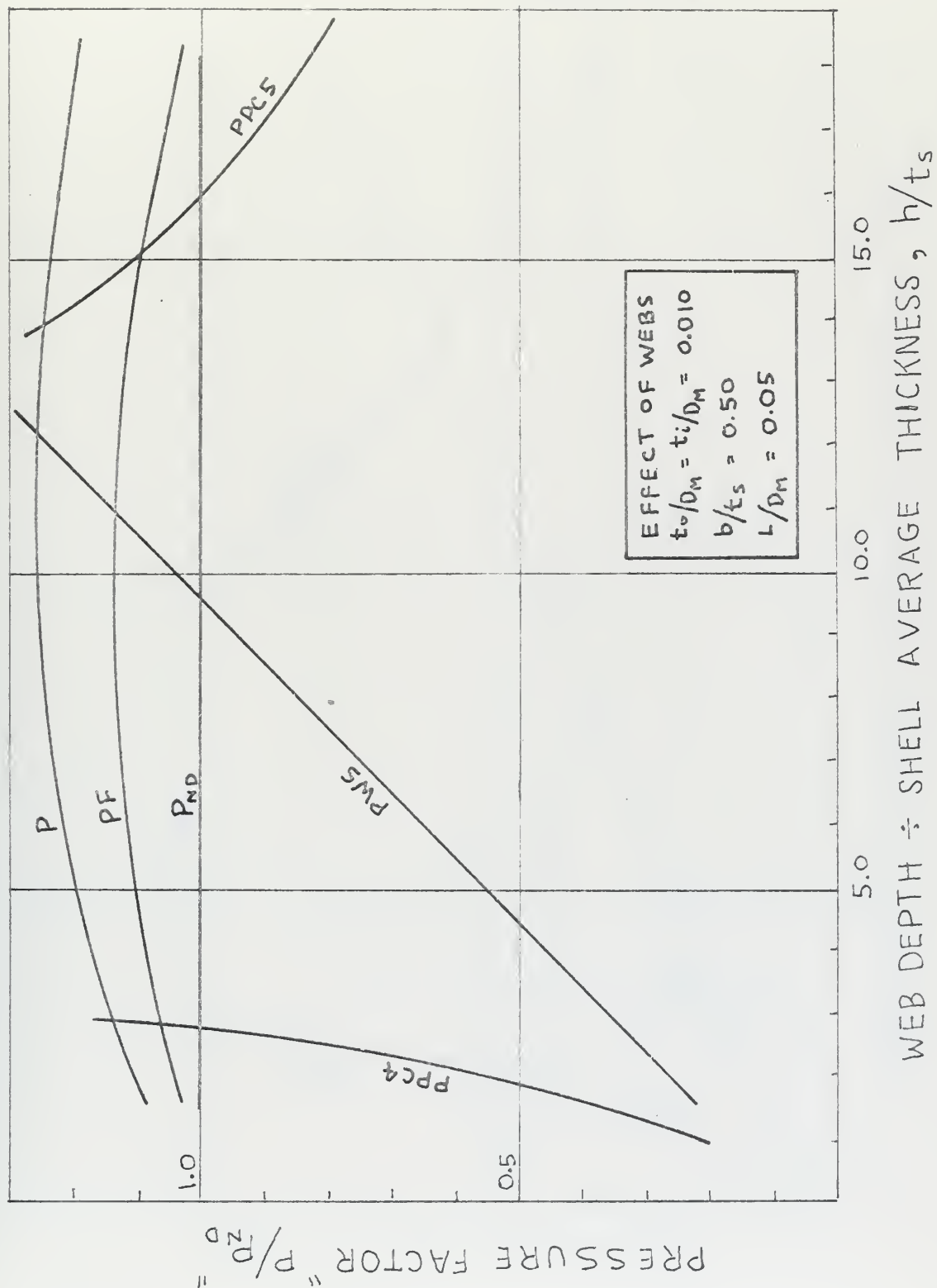
UNSUPPORTED LENGTH ÷ DIAMETER (L/D_m)



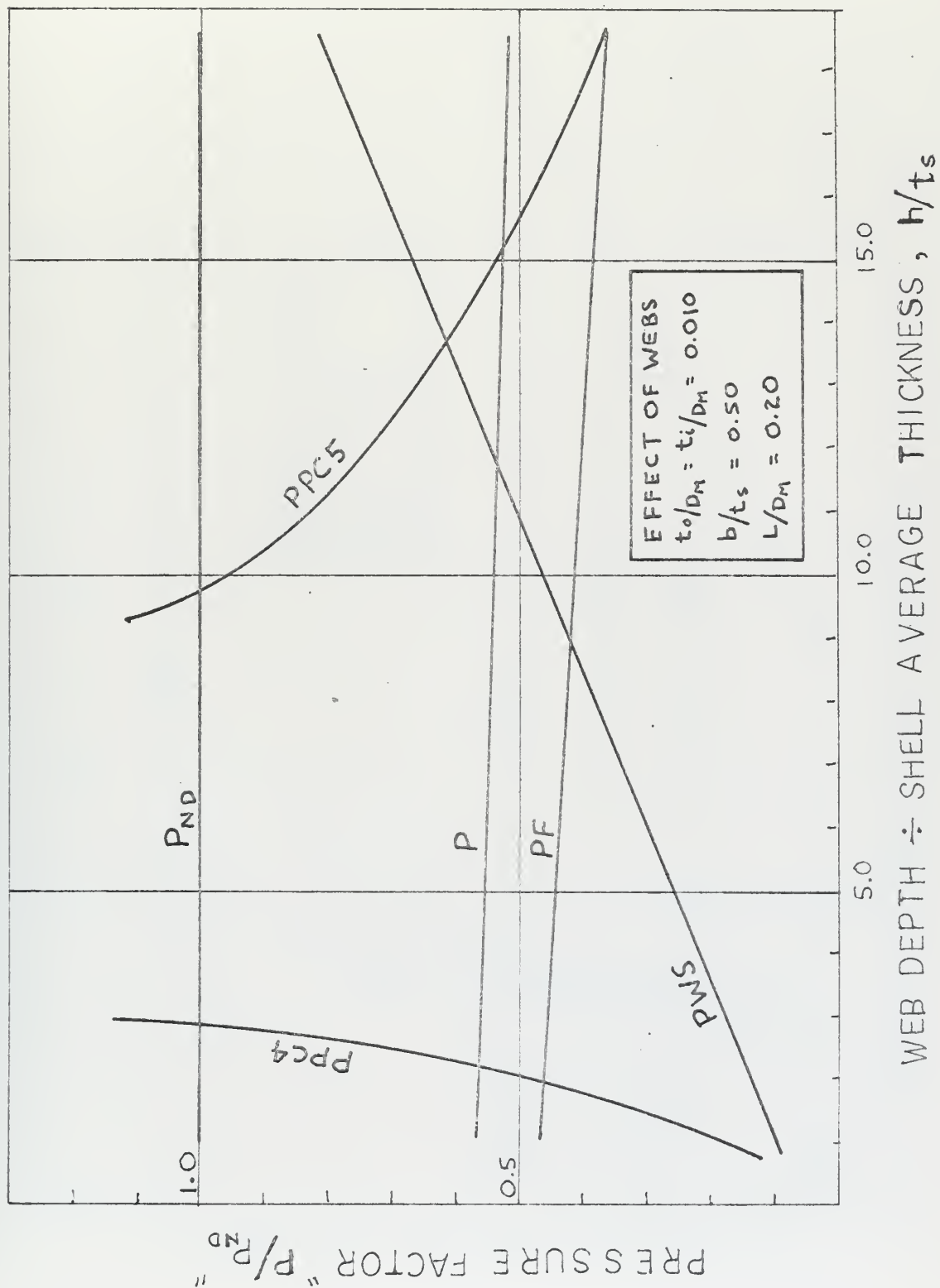


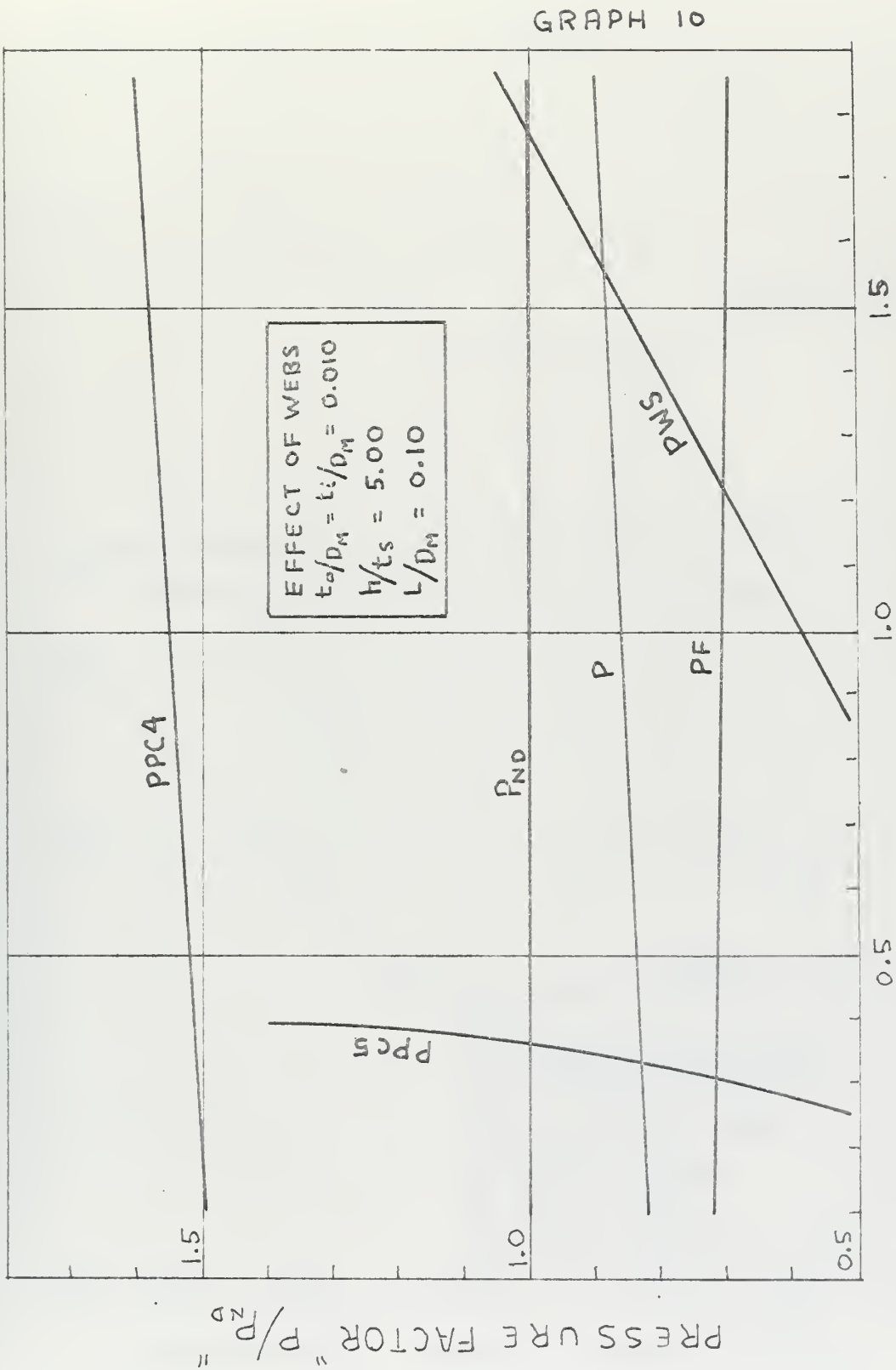


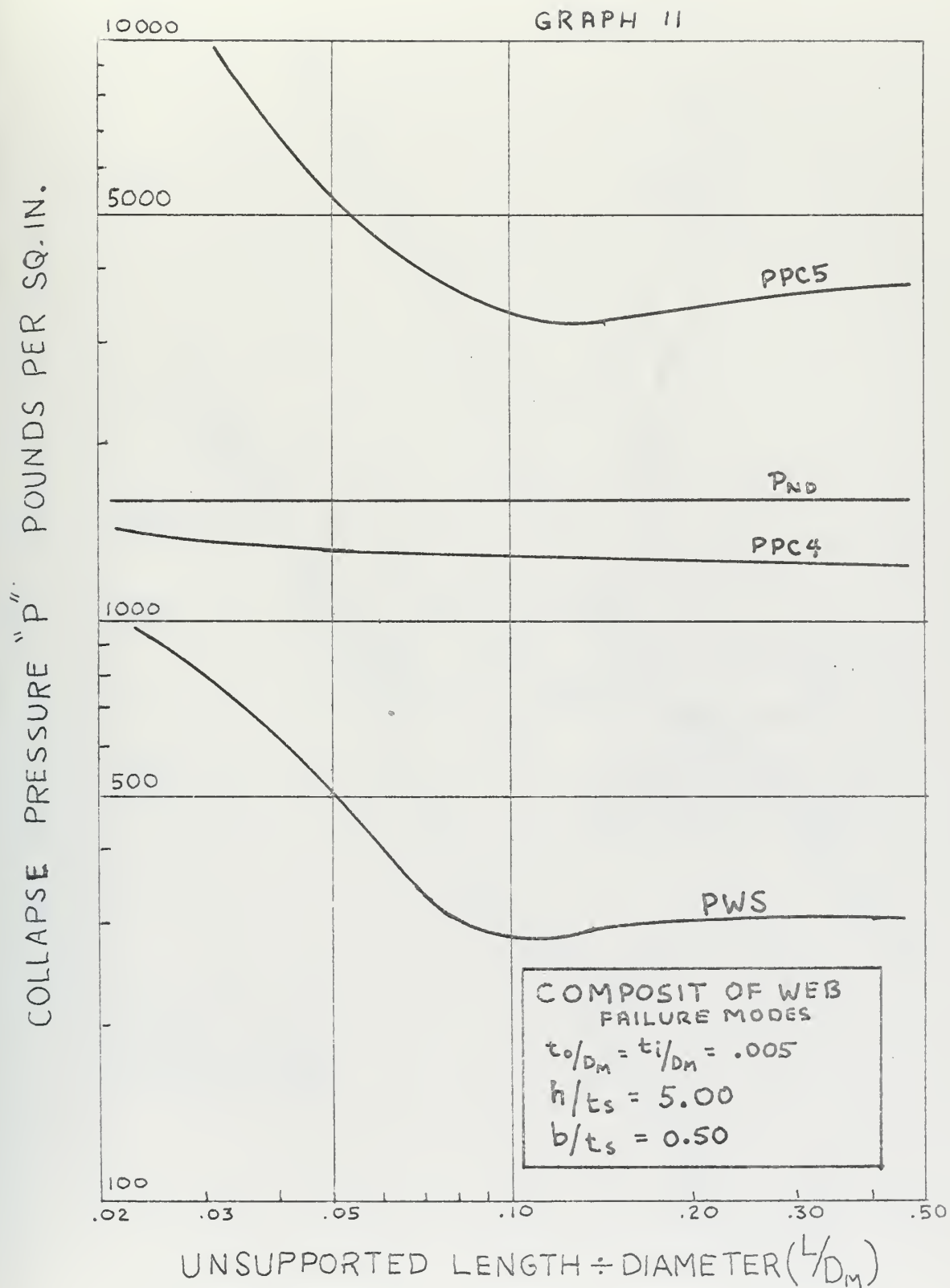
GRAPH 8

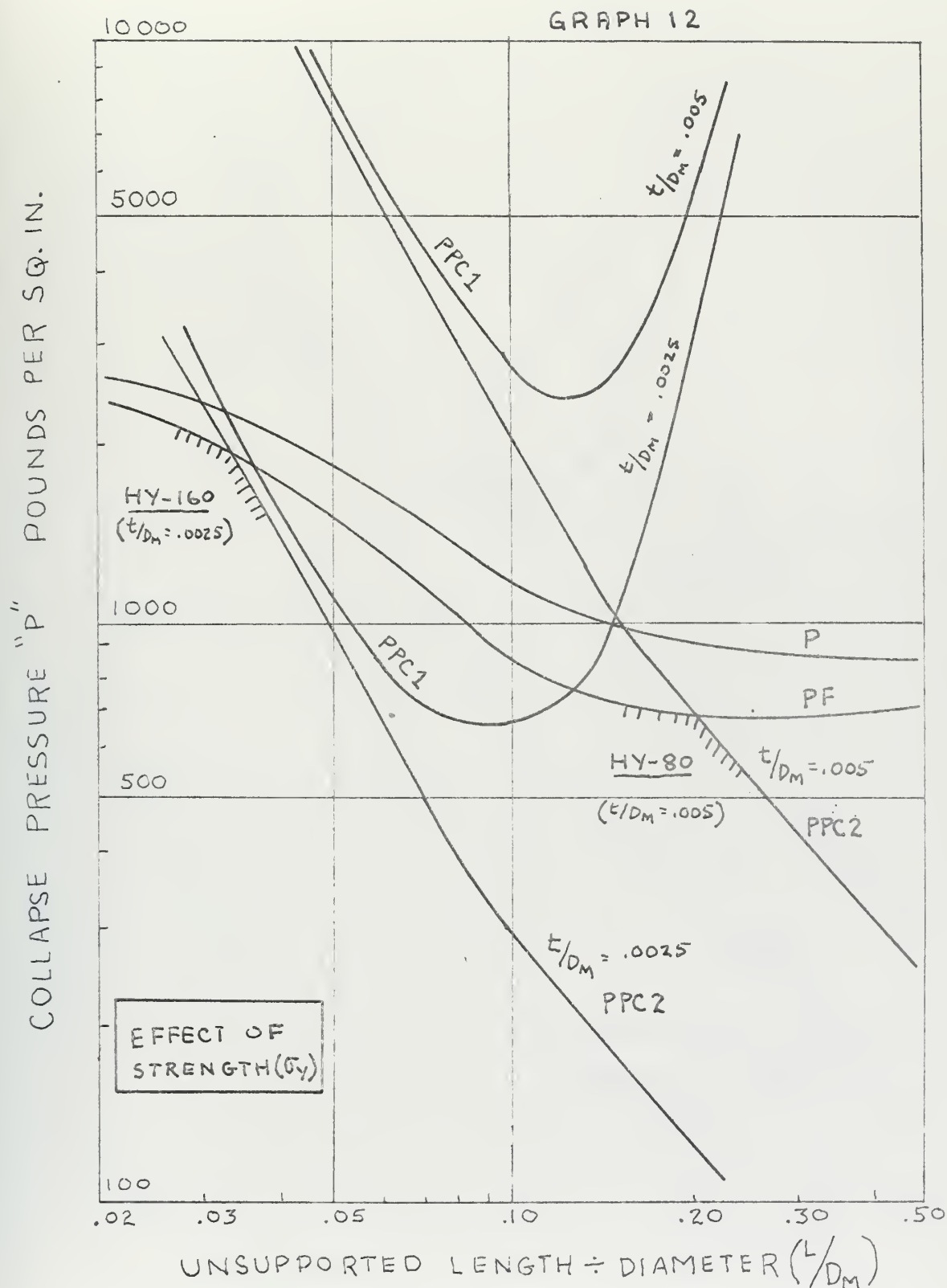


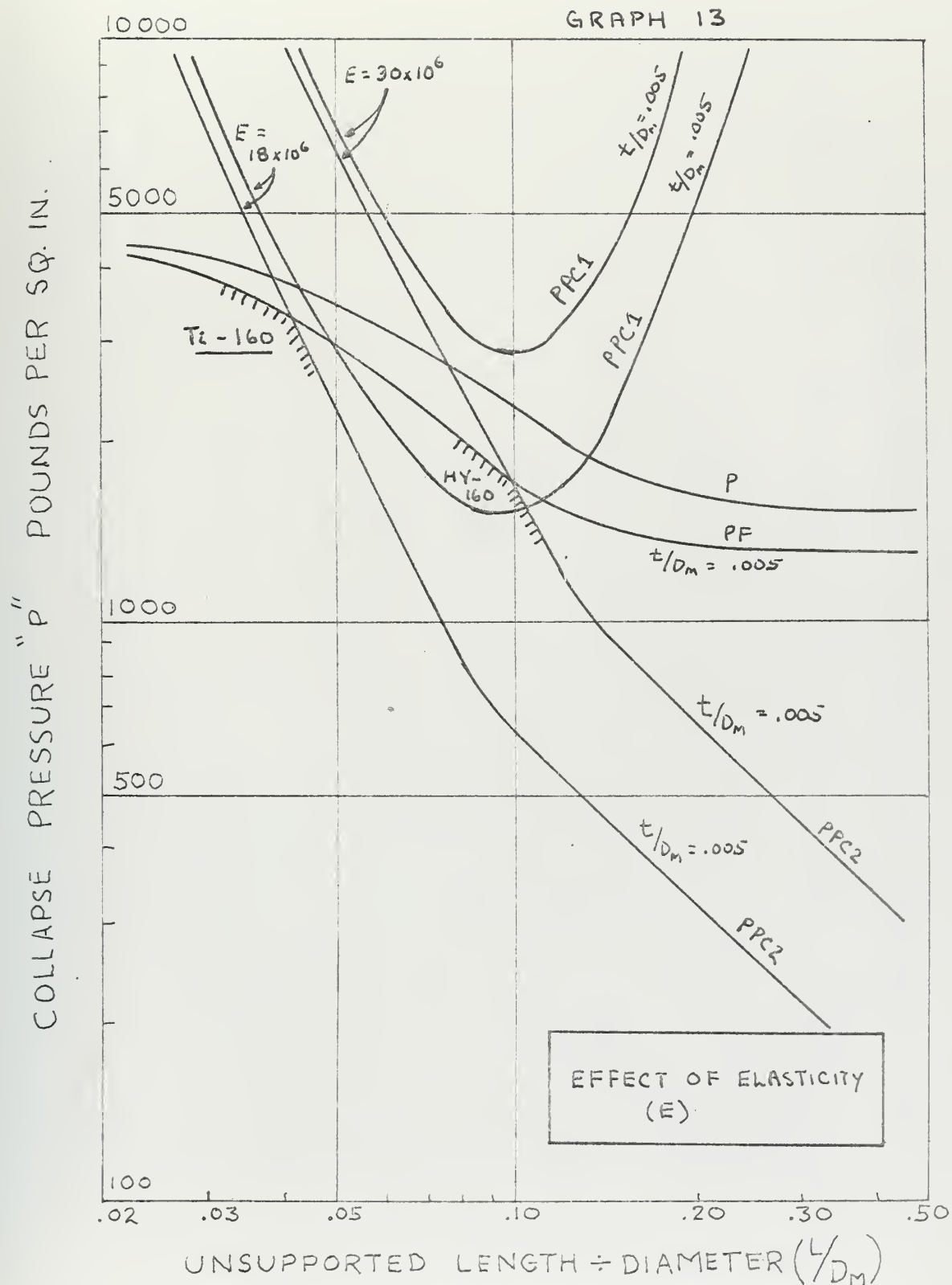
GRAPH 9



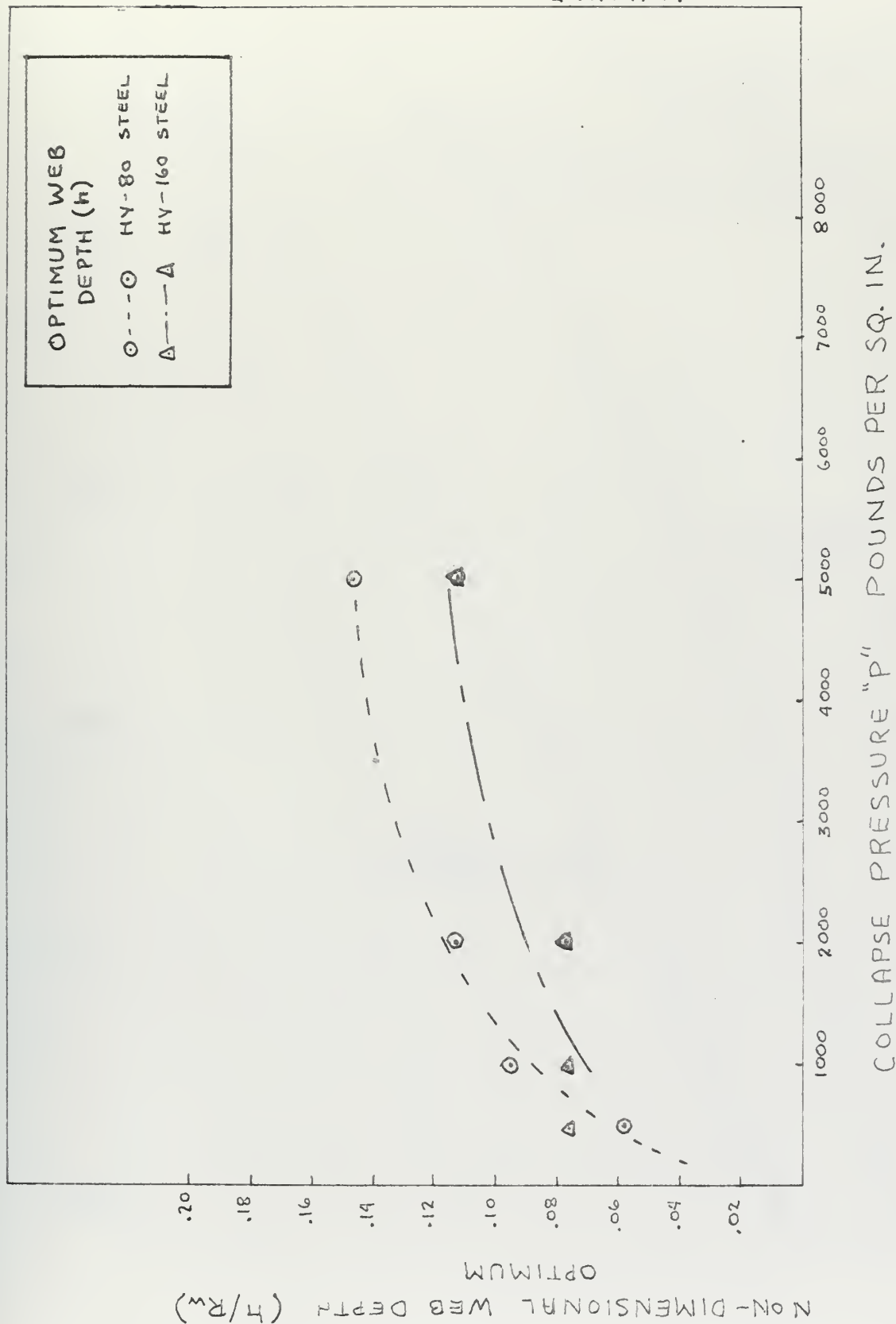








GRAPH 14



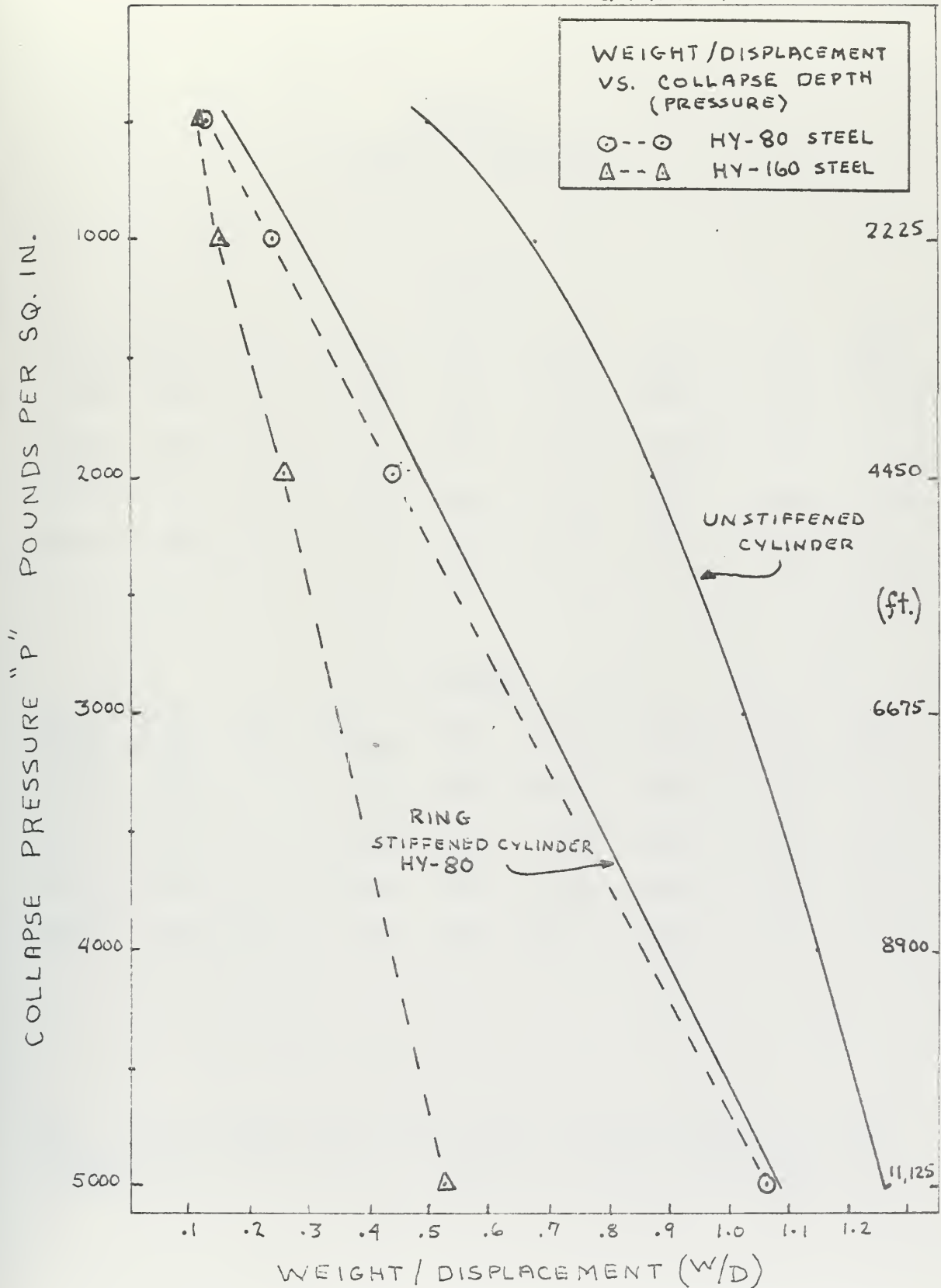


Table 1
OPTIMUM SCANTLINGS

HY-80

<u>P_{coll}</u>	<u>h/R_w</u>	<u>h/s</u>	<u>h/t_s</u>	<u>b/t_s</u>	<u>t_o/D</u>	<u>t_i/D</u>	<u>L/D</u>	<u>W/D</u>
500	.058	1.43	17.65	.588	.0019	.0015	.020	.137*
1000	.095	1.56	17.39	.696	.003	.00275	.030	.239
2000	.113	2.14	12.00	.600	.005	.005	.025	.436
5000	.148	2.16	6.15	.538	.014	.014	.030	1.061

HY-150

<u>P_{coll}</u>	<u>h/R_w</u>	<u>h/s</u>	<u>h/t_s</u>	<u>b/t_s</u>	<u>t_o/D</u>	<u>t_i/D</u>	<u>L/D</u>	<u>W/D</u>
500	.077	2.50	39.02	.976	.0011	.0010	.015	.125
1000	.077	1.87	26.67	.933	.0016	.0014	.020	.155
2000	.077	1.78	14.29	.893	.0028	.0028	.020	.258
5000	.113	2.14	9.23	.462	.008	.005	.025	.522

* This value seemed high and thus a scatter check was conducted about this point. Results of this check are in Table 2.

Table 2
OPTIMUM SCATTER ABOUT

$$P_{coll} = 500 \text{ psi}$$

<u>P_{coll}</u>	<u>Mode</u>	<u>h/R_N</u>	<u>t_o/b</u>	<u>t_i/D</u>	<u>L/D</u>	<u>b/t_s</u>	<u>W/D</u>	<u>W/D*</u>
508	PF _O	.058	.0014	.0014	.025	.786	.124	.122
554	PWS	.058	.0014	.0014	.020	.643	.128	.115
524	PWS	.058	.0015	.0014	.025	.759	.127	.121
581	PWS	.058	.0016	.0014	.020	.600	.134	.115
647	PPC4	.058	.0018	.0014	.020	.625	.145	.112
677	PPC4	.058	.0018	.0015	.020	.667	.153	.113

* Normalized W/D by $(\frac{500}{P_{coll}})$.

DISCUSSION OF RESULTS

1. High strength materials having inelastic zones are more prone to failure by buckling than elastic materials because of the reduction of the value of E above the proportional limit.
2. Graphs 2 through 5 indicate that close spacing (small L) of the annular rings (webs) is required in order to utilize the benefits of this type of structure. This follows from the more uniform stressing in this configuration.
3. Maximum stresses in the shell occur in the outer shell ($T_o = T_i$) when web spacing (L) is large. However, when the spacing is small the stress in the inner shell can exceed the outer. This occurs when the web thickness and web depth is insufficient to carry the load (see Graphs 6 and 7). Graph 6 suggests that a more evenly distributed stress can be obtained by making $T_o > T_i$ for thin shells. Optimization bears this out (Table 1).
4. Graphs 8 through 11 indicate that web strength and buckling determine web depth and thickness requirements and have a critical effect on the weight/displacement ratio ($PWS = f(bh)$, $PPC4 = f(bh^3)$, $PPC5 = f(b/h)^2$).

5. Shell buckling modes (PPC1, PPC2) become prevalent only in (a) thin shells -- either shallow depth load design or high strength materials, or (b) materials with low moduli of elasticity (E) such as titanium or aluminum, or (c) in the inelastic range of high strength materials.
6. Axisymmetric failure mode (PPC1) is prevalent only in very thin shells ($t/D_m < .0015$).
7. Higher strength materials and materials with low moduli of elasticity should have smaller web spacing in the optimum proportions (PPC2 shifts to the left -- thinner shells for the high strength materials; smaller E for those with low modulus of elasticity).
8. Table 1 and Graph 14 indicate that web depth increases with depth for the optimum scantlings. This is seen from the need to increase the web size with loading. Also h/t_s decreases with depth indicating t_s is increasing faster than web depth, h . The parameters h/s and b/t_s do not change appreciably.
9. Optimization scatter about the optimum scantlings indicates more than one optimum configuration (Table 2).
10. Graph 15 shows that there is an advantage of this structure over the ring-stiffened cylinder of some order. Further examination is required using membrane yield in

place of yielding at the outer fibers to compare to the optimized results of ring-stiffened cylinders using Lunchick's formulation for shell yield. Also, comparison with experimental failures is required to verify the formulation used in this investigation or to justify the use of membrane stresses.

11. The advantages of thinner shells (homogeneity of material, greater ductility, better notch toughness, and easier to form or weld) compared to those of the ring-stiffened cylinder make the web-core sandwich cylindrical hull very appealing.

12. The results set forth are thought to be conservative. Yield (P , PF , PWS) at the outer fibers was the criteria of yield failure. If membrane stresses were permitted to reach yielding more dramatic results would have ensued.

CONCLUSIONS

1. Web-core sandwich type cylindrical pressure hulls possess structural efficiencies in the order of 10-20% higher than those of conventional ring-stiffened construction at depths of 2,000 feet and 5% higher at depths of 10,000 feet.
2. Higher strengths realized with the sandwich designs over those of conventional ring-stiffened cylinders can be explained:
 - a. Symmetry of the sandwich cross section allows a more uniform and more complete stressing of the available material. The outstanding flange of a T-frame on a conventional ring-stiffened pressure hull is stressed only in the circumferential direction, i.e., a uniaxial state of stress exists. In a sandwich pressure hull, the "flange" material is in reality a second shell which is more efficiently stressed in both the axial and circumferential directions, i.e., a biaxial state of stress exists.
2. Sandwich geometry is inherently more stable structurally permitting straining of the material well into the work-hardening range with concomitant beneficial effects of higher strength levels (inelastic materials).

3. Inelastic analysis is very necessary in the use of high strength materials.

4. Sandwich construction permits the use of thinner plating material with its superior homogeneous, higher strength, greater ductility, and better notch toughness characteristics over thicker plating. In addition, thinner plating is easier to form and weld.

5. The use of this type of structure is appealing in several designs:

- (a) Large diamters hulls -- thinner plating.
- (b) Deep depth hulls -- thinner plating.
- (c) Shallow depth hulls (<2000 feet) -- 10-20% more efficient structure.

RECOMMENDATIONS

1. Utilize the membrane stresses developed in Appendix A in the stress analysis and failure mode programs in place of the outer fiber stresses.
2. Compare experimental results to theoretical and determine whether membrane stresses or outer fiber stresses are more realistic.
3. Formulate a more sophisticated optimization program utilizing a search technique (reduce program cost).

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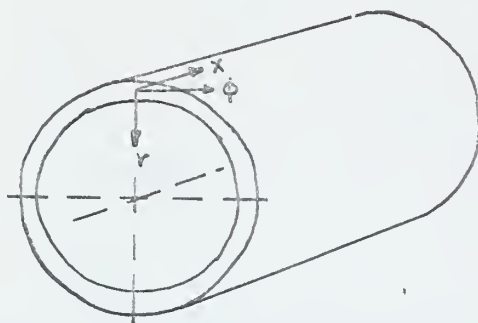
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APPENDIX A

AXISYMMETRIC STRESSES IN AN ISOTROPIC, WEB-STIFFENED SANDWICH CYLINDER LOADED WITH UNIFORM EXTERNAL PRESSURE

BASIC EQUATIONS

For cylindrical shells loaded under uniform pressure, the three principal axes of strain are



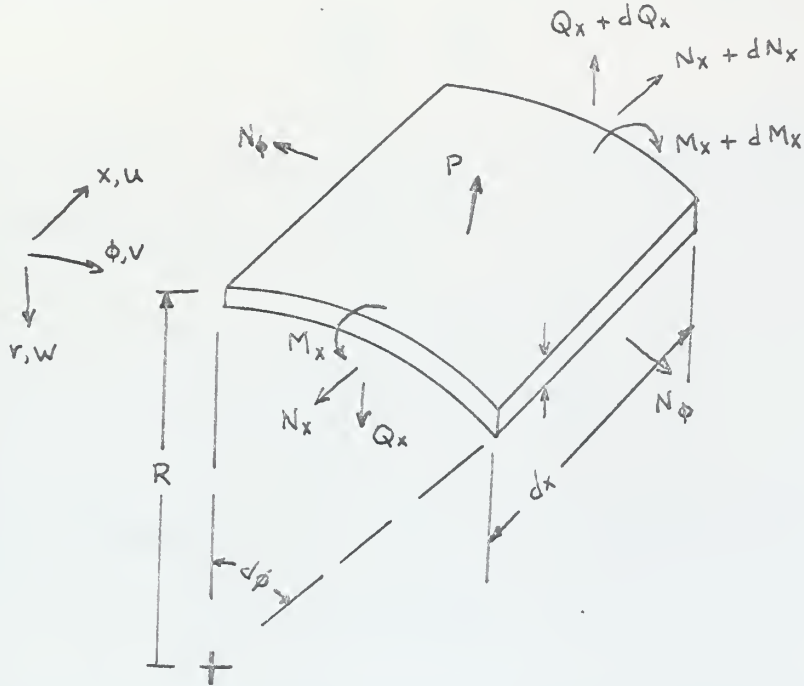
$$\begin{aligned}\epsilon_x &= \frac{1}{E} \left[\sigma_x - \nu (\sigma_\phi + \sigma_r) \right] \\ \epsilon_\phi &= \frac{1}{E} \left[\sigma_\phi - \nu (\sigma_x + \sigma_r) \right] \\ \epsilon_r &= \frac{1}{E} \left[\sigma_r - \nu (\sigma_x + \sigma_\phi) \right]\end{aligned}\tag{1}$$

Since the radial stress in the shell is generally small in comparison to the longitudinal and circumferential stresses, Equation (1) reduces to

$$\begin{aligned}\epsilon_x &= \frac{1}{E} \left[\sigma_x - \nu \sigma_\phi \right] \\ \epsilon_\phi &= \frac{1}{E} \left[\sigma_\phi - \nu \sigma_x \right]\end{aligned}\tag{2}$$

for the longitudinal and circumferential strains, respectively.

The equilibrium of forces and moment of forces in the element requires that



$$\frac{dQ_x}{dx} - \frac{N_\phi}{R} = -P$$

(3)

$$\frac{dM_x}{dx} = Q_x$$

if the beam-column effect due to the axial portion of the hydrostatic pressure is neglected.

The moment curvature relationship

$$M_x = -D \frac{d^2 w}{dx^2}$$

where

$$D = \frac{Et^3}{12(1-\nu^2)} \quad (4)$$

Equations (3) and (4) can be combined to form the following general differential equation of equilibrium

$$D \frac{d^4 w}{dx^4} + \frac{N_\phi}{R} = P \quad (5)$$

The strains in equation (2) can be expressed in terms of the forces N_x and N_ϕ as

$$\begin{aligned} \epsilon_x &= \frac{1}{Et} (N_x - \nu N_\phi) \\ \epsilon_\phi &= \frac{1}{Et} (N_\phi - \nu N_x) \end{aligned} \quad (6)$$

The circumferential strain can also be expressed as

$$\epsilon_\phi = \frac{w}{R} \quad (7)$$

By substituting Equations (6) and (7) into Equation (5), the general equation of equilibrium is

$$D \frac{d^4 w}{dx^4} + \frac{Et}{R^2} w = P - \frac{\nu}{R} N_x \quad (8)$$

The edge conditions at $X = 0$ and $X = L$ are

$$\left. \frac{dw}{dx} \right|_{x=0} = \left. \frac{dw}{dx} \right|_{x=L} = 0$$

and

$$\left. \frac{-d^3 w}{dx^3} \right|_{x=0} = \left. \frac{d^3 w}{dx^3} \right|_{x=L} = \frac{Q_x}{D} \quad (9)$$

With these boundary conditions, the general solution of equation (8) is

$$\begin{aligned}
 w = & - \frac{2R^2 Q_x}{EtL} \frac{\theta}{2} \left(B_1 B_3 + B_4 - B_5 - B_2 B_6 \right) \\
 & + \frac{R^2 P}{Et} - \frac{\nu}{E} \frac{N_x R}{t}
 \end{aligned} \quad (10)$$

where

$$\theta = \sqrt[4]{\frac{3(1-\nu^2)}{t^2 R^2}} L$$

$$B_1 = \frac{\sinh \theta + \sin \theta}{\cosh \theta - \cos \theta}$$

$$B_2 = \frac{\sinh \theta - \sin \theta}{\cosh \theta - \cos \theta}$$

$$B_3 = \cos \left(\theta \frac{x}{L} \right) \cosh \left(\theta \frac{x}{L} \right)$$

$$B_4 = \sin \left(\theta \frac{x}{L} \right) \cosh \left(\theta \frac{x}{L} \right)$$

$$B_5 = \cos \left(\theta \frac{x}{L} \right) \sinh \left(\theta \frac{x}{L} \right)$$

$$B_6 = \sin \left(\theta \frac{x}{L} \right) \sinh \left(\theta \frac{x}{L} \right)$$

STRESSES AND STRAINS

The longitudinal stresses on the shell section with externally applied pressure for the outer and inner surfaces are

$$\sigma_x = \frac{N_x}{t} \pm \frac{6M_x}{t^2} \quad (11)$$

From Equations (4) and (10), the longitudinal stresses of Equation (11) are

$$\sigma_x = \frac{R}{t} \left[\frac{N_x}{R} + \frac{Q_x G_1}{L\psi} \right] \quad (12)$$

From Equations (2), (7), (10), and (12), the circumferential stresses are

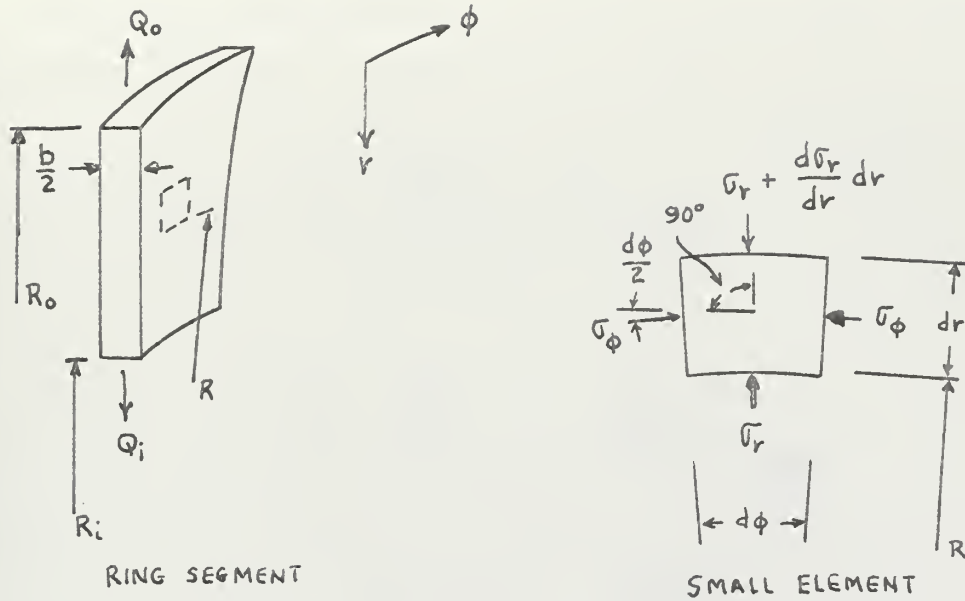
$$\sigma_\phi = - \frac{R}{t} \left[-P + \frac{Q_x}{L} \left(G_2 \pm \nu \frac{G_1}{\psi} \right) \right] \quad (13)$$

where

$$\begin{aligned} G_1 &= \theta (B_2 B_3 - B_4 - B_5 + B_1 B_6) \\ G_2 &= \theta (B_1 B_3 + B_4 - B_5 - B_2 B_6) \\ \psi &= \sqrt{\frac{(1 - \nu^2)}{3}} \end{aligned} \quad (14)$$

DEFORMATION OF ISOTROPIC CIRCULAR RINGS

For a web-stiffened sandwich cylinder, the outside and inside shells are separated by a circular ring. A segment for this type of ring is shown below.



Neglecting stresses in the X-direction for the ring,
Equation (1) reduces to

$$\begin{aligned}\epsilon_\phi &= \frac{1}{E} \left(\sigma_\phi - \nu \sigma_r \right) \\ \epsilon_r &= \frac{1}{E} \left(\sigma_r - \nu \sigma_\phi \right)\end{aligned}\tag{15}$$

These strains can also be expressed as

$$\begin{aligned}\epsilon_\phi &\approx \frac{w}{R} \\ \epsilon_r &= \frac{dw}{dr}\end{aligned}\tag{16}$$

The equilibrium of radial forces on the small element requires that

$$\sigma_\phi - \sigma_r - R \frac{d\sigma_r}{dr} = 0\tag{17}$$

With the relationships in Equations (15) and (16), the general solution of Equation (17) is

$$\begin{aligned}
 W(R) = & \frac{\left[-Q_i \left(\frac{RRI}{RRO} \right)^2 + Q_o \right] \left(\frac{R}{RRO} \right)}{(1 - \nu)} \\
 & + \frac{\left[-Q_i \left(\frac{RRI}{RRO} \right) + Q_o \left(\frac{RRI}{RRO} \right) \right] \left(\frac{RRI}{R} \right)}{(1 - \nu)} \\
 & \frac{Z}{2} \left(\frac{b}{RRO} \right) \left[1 - \left(\frac{RRI}{RRO} \right)^2 \right]
 \end{aligned} \tag{18}$$

where

$$\begin{aligned}
 Z &= \frac{E}{1 - \nu^2} \\
 RRO &= RO + \frac{TO}{2} \\
 RRI &= RI - \frac{TI}{2} \\
 RR &= \frac{RRI}{RRO}
 \end{aligned}$$

By differentiating Equation (18) and by combining Equations (15), (16), and (17), the circumferential stress is

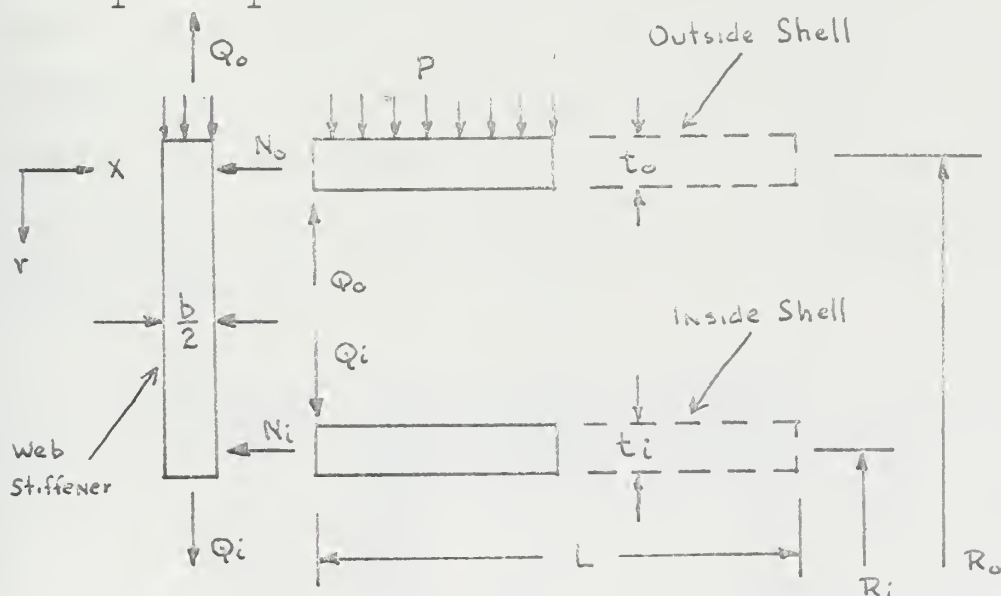
$$\sigma_{\phi}(R) = \frac{\left[-Q_i (RR)^2 + Q_o \right] \left(\frac{R}{RRO} \right) + \left[-Q_i (RR) + Q_o (RR) \right] \left(\frac{RRI}{R} \right)}{\frac{b}{2} \left(\frac{R}{RRO} \right) \left[1 - (RR)^2 \right]} \tag{19}$$

and the radial stress is

$$\sigma_r(R) = \frac{\left[-Q_i (RR)^2 + Q_o \right] \left(\frac{R}{RRO} \right) - \left[-Q_i (RR) + Q_o (RR) \right] \left(\frac{RRI}{R} \right)}{\frac{b}{2} \left(\frac{R}{RRO} \right) \left[1 - (RR)^2 \right]} \quad (20)$$

DEFORMATIONS OF SANDWICH SHELLS

Before determining the stresses in the cylindrical shells of a sandwich structure, it is first necessary to obtain the forces Q_o and N_o on the outside shell and the forces Q_i and N_i on the inside shell.



The radial deflection of the outside shell at the web-shell juncture can be expressed in terms of the forces as

$$w_{os} = g_o Q_o + d_o N_o + f_o P \quad (21)$$

and the radial deflection of the inside shell

$$W_{is} = g_i Q_i + d_i N_i \quad (22)$$

Similarly, the radial deflection of the outer fiber of the web at the web-shell juncture can be expressed as

$$W_{ow} = g_{oo} \left[Q_o + \frac{b}{2} p \right] + g_{oi} Q_i \quad (23)$$

and the radial deflection of the inner fiber can be expressed as

$$W_{iw} = g_{io} \left[Q_o + \frac{b}{2} p \right] + g_{ii} Q_i \quad (24)$$

Since the stresses in a sandwich structure are linear functions of the applied pressure, the coefficients of the force in Equations (21), (22), (23), and (24) will be developed for a unit of applied pressure ($p = 1.0$). By substituting the geometric properties of the outside shell into Equation (10), the coefficients of the forces in Equation (21) are

$$g_o = - \frac{2RO^2 \theta_o}{2ETOL} \left[\frac{\sinh \theta_o + \sin \theta_o}{\cosh \theta_o - \cos \theta_o} \right] \quad (25)$$

$$d_o = - \frac{vRO}{ETO}$$

$$f_o = \frac{RO^2}{ETO}$$

where

$$\theta_o = \frac{\frac{L}{2}}{\sqrt{3(1-v^2)} \sqrt{RO \cdot TO}}$$

Also by the geometric properties of the inside shell, the coefficients of the forces in Equation (22) are

$$g_i = \frac{-2RI^2\theta_i}{2ETIL} \left[\frac{\sinh\theta_i + \sin\theta_i}{\cosh\theta_i - \cos\theta_i} \right]$$

$$d_i = -\frac{\nu RI}{ETI}$$
(26)

where

$$\theta_i = \frac{4}{\sqrt{3(1-\nu^2)}} \frac{L}{\sqrt{RI \cdot TI}}$$

The hyperbolic and trigonometric function in Equations (25) and (26) are redefined by

$$F \equiv \frac{\theta}{2} \left[\frac{\sinh\theta + \sin\theta}{\cosh\theta - \cos\theta} \right]$$
(27)

By Equation (27), the coefficients g_o and g_i are

$$g_o = \frac{-2RO^2}{ETOL} F_o$$

$$g_i = \frac{-2RI^2}{ETIL} F_i$$
(28)

By substituting combinations of unit and zero forces into Equation (17), the coefficients in Equations (23) and (24) are

$$g_{oo} = CRRO [(1 + RR^2) - \nu(1 - RR^2)]$$

$$g_{oi} = -2CRRI(RR)$$

$$g_{io} = 2CRRO(RR)$$

$$g_{ii} = -CRRI [(1 + RR^2) - \nu(1 - RR^2)]$$
(29)

where

$$C = \frac{2}{bE(1 - RR^2)}$$

From the equilibrium conditions of the end forces on a sandwich cylinder loaded under an external uniform pressure, the relationship between the outside and inside axial forces for $p = 1.0$ psi is

$$N_O R_O + N_I R_I = \frac{RO^2}{2} \quad (30)$$

By assuming that the axial membrane strain in the outside shell is equal to the axial membrane strain in the inside shell, i.e., $\frac{N_O}{ET_O} = \frac{N_I}{ET_I}$, the axial forces are

$$N_O = \frac{\frac{RO}{2}}{1 + \frac{TI}{TO} \cdot \frac{RI}{RO}} \quad (31)$$

$$N_I = \frac{TI}{TO} N_O$$

For compatibility of deformations in the radial direction, the relationships that must exist are $W_{OS} = W_{OW}$ and $W_{IS} = W_{IW}$. With these relationships and Equations (21) to (29), the radial forces Q_O and Q_I can be expressed in terms of the two following simultaneous equations:

$$\begin{aligned}
(g_o - g_{oo}) Q_o - g_{oi} Q_i &= \frac{-RO^2}{ETO} \left[1 - \frac{v/2}{1 + \frac{TI \cdot RI}{TO \cdot RO}} \right] \\
&\quad + \frac{b}{2} g_{oo} \\
-g_{io} Q_o + (g_i - g_{ii}) Q_i &= \frac{v}{2E} \frac{RO^2}{TI} \left[1 - \frac{1}{1 + \frac{TI \cdot RI}{TO \cdot RO}} \right] \\
&\quad + \frac{b}{2} g_{io}
\end{aligned} \tag{32}$$

The forces N_o , N_i , Q_o , and Q_i can be computed from Equations (31) and (32).

STRESS DISTRIBUTION IN SANDWICH STRUCTURES

The stresses on the outside shell of the isotropic sandwich are determined by substituting N_o , Q_o , and the geometric properties of the outside shell into Equations (12) and (13). From these relationships, the circumferential stress in the outside shell on an outer fiber is

$$\sigma_{\phi oo} = -P \frac{RO}{TO} \left[-1 + \frac{Q_o}{L} (G_{20} + \frac{v}{\psi} G_{10}) \right]$$

on an inner fiber is

$$\sigma_{\phi oi} = -P \frac{RO}{TO} \left[-1 + \frac{Q_o}{L} (G_{20} - \frac{v}{\psi} G_{10}) \right]$$

and the membrane stress is

$$\sigma_{\phi om} = -P \frac{RO}{TO} \left[-1 + \frac{Q_o}{L} G_{20} \right] \tag{33}$$

The longitudinal stress in the outside shell on an outer fiber is

$$\sigma_{xoo} = P \frac{RO}{TO} \left[\frac{N_o}{RO} - \frac{Q_o}{L\psi} G_{10} \right]$$

on an inner fiber

$$\sigma_{xoi} = P \frac{RO}{TO} \left[\frac{N_o}{RO} + \frac{Q_o}{L\psi} G_{10} \right]$$

and the membrane stress is

(34)

$$\sigma_{xom} = P \frac{N_o}{TO}$$

And likewise the stresses on the inside shell are determined by substituting N_i , Q_i , and the geometric properties of the inside shell into Equations (12) and (13).

$$\sigma_{\phi io} = - P \frac{RI}{TI} \frac{Q_i}{L} \left[G_{2i} + \frac{\nu}{\psi} G_{1i} \right]$$

$$\sigma_{\phi ii} = - P \frac{RI}{TI} \frac{Q_i}{L} \left[G_{2i} - \frac{\nu}{\psi} G_{1i} \right]$$

$$\sigma_{\phi im} = - P \frac{RI}{TI} \frac{Q_i}{L} G_{2i} \quad (35)$$

$$\sigma_{xio} = P \frac{RI}{TI} \left[\frac{N_i}{RI} - \frac{Q_i}{L\psi} G_{1i} \right]$$

$$\sigma_{xii} = P \frac{RI}{TI} \left[\frac{N_i}{RI} + \frac{Q_i}{L\psi} G_{1i} \right]$$

$$\sigma_{xim} = P \frac{N_i}{TI} \quad (36)$$

The stresses in the web stiffeners at the outside and inside shell locations are determined by substituting the outside radius RR_O and inside radius RR_I into Equations (19) and (20). Also, the total force on the outer surface of the web consists of the radial force Q_O at the web-shell juncture and the force attributed to the applied pressure directly on the web. The total forces (H_O and H_I) on the outer and inner surfaces of the web for a unit pressure ($p = 1.0$ psi) are

$$\begin{aligned} H_O &= Q_O + b/2 \\ H_I &= Q_I \end{aligned} \quad (37)$$

With these relationships, the circumferential stress in the web on the outer surface is

$$\sigma_{\phi WO} = -P \frac{\left[\left(-H_I (RR)^2 + H_O \right) + \left(-H_I (RR) + H_O (RR) \right) (RR) \right]}{\frac{b}{2} (1 - RR^2)}$$

and the radial stress (38)

$$\sigma_{rWO} = -P \frac{\left(-H_I (RR)^2 + H_O \right) - \left(-H_I (RR) + H_O (RR) \right) (RR)}{\frac{b}{2} (1 - RR^2)}$$

and on the inner surface

$$\sigma_{\phi wi} = \frac{\left[\left(-H_i (RR)^2 + H_o \right) (RR) - \left(-H_i (RR) + H_o (RR) \right) \right]}{\frac{b}{2} (RR) (1 - RR^2)}$$

$$\sigma_{rwi} = \frac{\left[\left(-H_i (RR)^2 + H_o \right) (RR) - \left(-H_i (RR) + H_o (RR) \right) \right]}{\frac{b}{2} (RR) (1 - RR^2)}$$

(39)

APPENDIX B

BUCKLING ANALYSIS1. Inelastic Asymmetric (Lobar) Buckling Analysis.

The buckling equations for a fully plastic cylinder specialized for the case of hydrostatic pressure loading

$$\frac{E_T}{2E_S} \left(\frac{1}{2} \frac{\partial^2 u}{\partial x^2} + \frac{1}{R} \frac{\partial^2 v}{\partial x \partial \phi} + \frac{1}{R} \frac{\partial w}{\partial x} \right) \quad (1a)$$

$$+ \frac{3}{4} \frac{\partial^2 u}{\partial x^2} + \frac{1}{4R^2} \frac{\partial^2 u}{\partial \phi^2} + \frac{1}{4R} \frac{\partial^2 v}{\partial x \partial \phi} = 0$$

$$\frac{E_T}{E_S} \left(\frac{1}{2} \frac{\partial^2 u}{\partial x \partial \phi} + \frac{1}{R^2} \frac{\partial^2 v}{\partial \phi^2} + \frac{1}{R^2} \frac{\partial w}{\partial \phi} \right) \quad (1b)$$

$$+ \frac{1}{4} \frac{\partial^2 v}{\partial x^2} + \frac{1}{4R} \frac{\partial^2 u}{\partial x \partial \phi} = 0$$

$$\begin{aligned} & \frac{4}{3} \frac{E_S t}{R} \left(\frac{E_T}{E_S} \right) \left(\frac{1}{2} \frac{\partial u}{\partial x} + \frac{1}{R} \frac{\partial v}{\partial \phi} + \frac{w}{R} \right) \\ & + D \left[\frac{E_T}{E_S} \left(\frac{1}{4} \frac{\partial^4 w}{\partial x^4} + \frac{1}{R^2} \frac{\partial^4 w}{\partial x^2 \partial \phi^2} + \frac{1}{R^4} \frac{\partial^4 w}{\partial \phi^4} \right) \right. \\ & \left. + \frac{3}{4} \frac{\partial^4 w}{\partial x^4} + \frac{1}{R^2} \frac{\partial^4 w}{\partial x^2 \partial \phi^2} \right] + N_x \frac{\partial^2 w}{\partial x^2} \\ & + N_\phi \frac{1}{R^2} \frac{\partial^2 w}{\partial \phi^2} + P = 0 \end{aligned} \quad (1c)$$

where x and ϕ are respectively the axial and circumferential coordinates,

u , v , and w are the axial, tangential, and radial displacements,

E_S and E_T are the secant and tangent moduli,

R is the radius to shell mid-surface,

t is the shell thickness,

ν is Poisson's ratio,

D is the bending rigidity $= E_S t^3 / 12(1-\nu^2)$

N_x and N_ϕ are forces per unit length in the axial and circumferential directions, and

p is the hydrostatic pressure.

With several differentiations, u and v can be eliminated from Equations (1a), (1b) and (1c) so that a single eight-order equation in w is obtained:

$$\begin{aligned}
 D \left[\frac{E_T}{E_S} \nabla^8 w + \left(1 - \frac{E_T}{E_S} \right) \left\{ \nabla^4 \left(\frac{3}{2} \frac{\partial^4 w}{\partial x^4} + \frac{1}{R^2} \frac{\partial^4 w}{\partial x^2 \partial \phi^2} \right) \right. \right. \\
 \left. \left. + \frac{3}{4} \left(\frac{E_S}{E_T} - 1 \right) \left(\frac{3}{4} \frac{\partial^8 w}{\partial x^8} + \frac{1}{R^2} \frac{\partial^8 w}{\partial x^6 \partial \phi^2} \right) \right\} \right] \\
 + \frac{E_S t}{R^2} \frac{\partial^4 w}{\partial x^4} + N_x \left[\nabla^4 \frac{\partial^2 w}{\partial x^2} + \frac{3}{4} \left(\frac{E_S}{E_T} - 1 \right) \frac{\partial^6 w}{\partial x^6} \right] \\
 + N_\phi \left[\nabla^4 \frac{1}{R^2} \frac{\partial^2 w}{\partial \phi^2} + \frac{3}{4} \frac{1}{R^2} \left(\frac{E_S}{E_T} - 1 \right) \frac{\partial^6 w}{\partial x^4 \partial \phi^2} \right] = 0
 \end{aligned} \tag{2}$$

where ∇^4 indicates $\left[\frac{\partial^2}{\partial x^2} + \frac{1}{R^2} \frac{\partial^2}{\partial \phi^2} \right]^2$

A solution to this equation can be written:

$$W = A \sin K\phi \sin \lambda x \quad (3)$$

where $K = N$

$$\lambda = \frac{m\pi}{L}$$

L is the length of the shell, and

m and n are integers.

This solution satisfies the conditions of simple support at the ends of the cylinder, i.e., that w and $\frac{\partial^2 w}{\partial x^2}$ vanish at $x = 0$ and $x = 1$.

These conditions are not unreasonable for stiffened cylinders since it is likely that the effective rotational restraint will be limited by the formation of plastic regions arising from high bending stresses near stiffeners.

By substituting the solution (3) back into (2), the following characteristic-value equation is obtained:

$$D \left[\frac{E_T}{E_S} \left(\frac{K^2}{R^2} + \lambda^2 \right)^4 + \left(1 - \frac{E_T}{E_S} \right) \lambda^2 \left\{ \left(\frac{K^2}{R^2} + \lambda^2 \right)^2 \right. \right. \\ \left. \left. \left(\frac{3\lambda^2}{2} + \frac{K^2}{R^2} \right) + \frac{3\lambda^4}{4} \left(\frac{E_S}{E_T} - 1 \right) \left(\frac{3\lambda^2}{4} + \frac{K^2}{R^2} \right) \right\} \right]$$

$$\begin{aligned}
& + \frac{E_s t}{R^2} \lambda^4 - N_x \left[\left(\frac{K^2}{R^2} + \lambda^2 \right)^2 \lambda^2 + \frac{3}{4} \left(\frac{E_s}{E_T} - 1 \right) \lambda^6 \right. \\
& \left. + \frac{N_\phi}{N_x} \frac{K^2}{R^2} \left\{ \left(\frac{K^2}{R^2} + \lambda^2 \right)^2 + \frac{3}{4} \left(\frac{E_s}{E_T} - 1 \right) \lambda^4 \right\} \right] = 0
\end{aligned} \tag{4}$$

To simplify:

$$\begin{aligned}
\phi & \equiv \frac{\lambda^2}{\lambda^2 + \frac{K^2}{R^2}} = \frac{1}{1 + \frac{N^2 L^2}{m^2 \pi^2 R^2}} \\
f & \equiv \frac{N_x}{N_\phi} \\
c & \equiv \left(\frac{E_s}{E_T} - 1 \right)
\end{aligned} \tag{5}$$

Now since: $N_x = \sigma_x t = K_x p t$

the equation is then rearranged so that an expression for P_p , the plastic buckling pressure, is obtained:

$$P_p = \frac{f D \lambda^2 \frac{E_T}{E_s} \left[1 + c \phi \left\{ 1 + \frac{\phi}{2} + \frac{3}{4} \phi^2 \left(1 - \frac{\phi}{4} \right) \right\} \right] + \frac{E_s t f \phi^4}{R^2 \lambda^2}}{K_x t \phi \left[1 - \phi (1 - f) \right] \left[1 + 3 \frac{c \phi^2}{4} \right]}$$

(6)

Equation (6) can now be minimized for P_p with respect to N/M by setting

$$\frac{\partial P_p}{\partial \phi} = 0 \quad (7)$$

$$\phi_p^4 = \frac{m^4 \pi^4}{9} \frac{E_T}{E_S} \left(\frac{\sqrt{Rt}}{L} \right)^4 \left[\frac{1 - 2\phi_p (1-f) + \frac{3}{4}c\phi_p^2}{3 - 2\phi_p (1-f) + \frac{3}{4}c\phi_p^2} \right] \quad (8)$$

and

$$P_p = \frac{4m^2 \pi^2 E_T f}{9\phi_p} \left(\frac{t}{RK_x} \right) \left(\frac{Rt}{L} \right)^2 \left[\frac{1 + \frac{c\phi_p}{4} (3 + \phi_p + \frac{3}{4}c\phi_p^2)}{3 - 2\phi_p (1-f) + \frac{3}{4}c\phi_p^2} \right] \quad (9)$$

Since P_p in (9) is proportional to m^2 , m must be equal to one in all cases when N is greater than 0.

After some investigation, an approximate value of ϕ was determined

$$\phi \approx 1.23 \frac{\sqrt{Rt}}{L} \quad (10)$$

Equation (9) can be further simplified to

$$P_p = \frac{4\pi^2 E_T f}{9\phi} \left(\frac{t}{RK_x} \right) \left(\frac{\sqrt{Rt}}{L} \right)^2 \left[\frac{1 + \frac{3c\phi}{4}}{3 - 2\phi(1-f)} \right] \quad (11)$$

A similar analysis for the elastic region yields:

$$P_e = \frac{\pi^2 E f}{3(1-\nu_c^2) \phi} \left(\frac{t}{RK_x} \right) \left(\frac{\sqrt{Rt}}{L} \right)^2 \left(\frac{1}{3-2\phi(1-f)} \right) \quad (12)$$

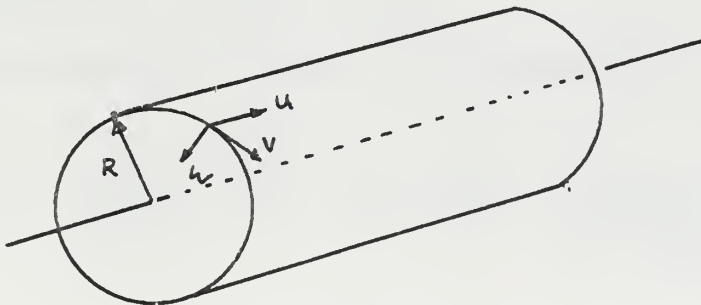
Although Equations (11), (12) define the buckling pressure for the plastic and elastic regions, no solution is given for the inelastic region which lies between these two limiting cases. However, by employing an empirical correction factor wherein Poisson's ratio is regarded as a variable, one can arrive at an expression which reduces to the proper limiting values.

$$\nu = \frac{1}{2} - \frac{E_s}{E} \left(\frac{1}{2} - \nu_e \right) \quad (13)$$

Thus rewriting Equation (11),

$$P_c = \frac{\pi^2 E_T f}{3(1-\nu^2)} \left(\frac{t}{RK_x} \right) \left(\frac{\sqrt{Rt}}{L} \right)^2 \left[\frac{1 + \frac{3\phi}{4} \left(\frac{E_s}{E_T} - 1 \right)}{3 - 2\phi(1-f)} \right] \quad (14)$$

2. Inelastic Axisymmetric Buckling Analysis*



The differential equations of equilibrium describing the plastic buckling of a cylindrical shell subjected to external hydrostatic pressure are expressed as Equations (1).

Equations (1) admits to a solution of the form

$$\begin{aligned} u &= A \cos N\phi \cos \lambda x \\ v &= B \sin N\phi \sin \lambda x \\ w &= c \cos N\phi \sin \lambda x \end{aligned} \quad (15)$$

where $\lambda = M\pi/L$, n an integer

This solution describes a buckling mode with M halfwaves along the cylinder axis and $2N$ halfwaves around its circumference. It also satisfies boundary conditions of simple support at the ends of the cylinder; i.e., w and $\partial^2 w / \partial x^2$ vanish at $x = 0$ and $x = L$.

The buckling condition of the shell is represented in Equation (4).

For axisymmetric buckling $N = 0$ and for $m = 1$, the minimum value of λ is $\lambda_{\min} = \pi/L$.

Substituting this value into Equation (4) results in

$$N_x = \frac{E_T t L^2}{9 \pi^2 R^2} \left[\frac{R^2 t^2 \pi^4 (1 + \frac{3}{4}c)}{L^4} + \frac{9(1+c)}{(1 + \frac{3}{4}c)} \right] \quad (16)$$

replacing N_x with $\sigma_x t = K_x p t$

$$P_p = \frac{E_T L^2}{9K_x \pi^2 R^2} \left[\frac{\pi^4 R^2 t^2 (1 + \frac{3}{4}c)}{L^4} + \frac{9(1+c)}{(1 + \frac{3}{4}c)} \right] \quad (17)$$

Again using the expression

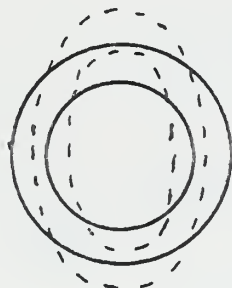
$$\nu = \frac{1}{2} - \frac{E_s}{E} \left(\frac{1}{2} - \nu_e \right)$$

$$P_c = \frac{E_T L^2}{12K_x (1-\nu^2) \pi^2 R^2} \left[\frac{\pi^4 R^2 t^2 (1 + \frac{3}{4}c)}{L^4} + \frac{12(1-\nu^2)(1+c)}{(1 + \frac{3}{4}c)} \right] \quad (18)$$

3. Web Buckling Under Hydrostatic Loading*

Two modes of buckling will be considered:

(a) Buckling of a Ring

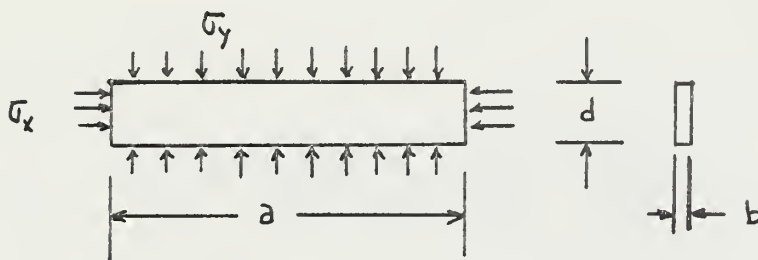


$N = 2$



$$P_c = (n^2 - 1) \frac{E_T I}{LR^3} = \frac{24 E_T I_{eff}}{D_m^3 L} \quad (19)$$

(b) Web Buckling under Edge Compression

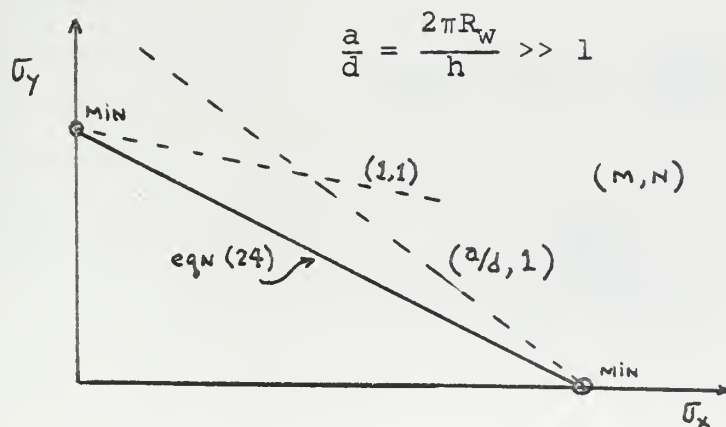


$$a = 2\pi R_w$$

$$d = h$$

$$\sigma_x m^2 + \sigma_y n^2 \left(\frac{a}{d}\right)^2 = \frac{\pi^2 D}{a^2 b} (m^2 + n^2 \frac{a^2}{d^2})^2 \quad (20)$$

where



$$\underline{\sigma_y = 0, m > 1, n = 1}$$

$$\sigma_x = \frac{\pi^2 a^2 D}{m^2 b} \left(\frac{m^2}{a^2} + \frac{n^2}{d^2} \right)^2 \rightarrow \frac{4\pi^2 D}{d^2 b} \quad (21)$$

$$\underline{\sigma_x = 0, m = 1, n = 1}$$

$$\sigma_y = \frac{\pi^2 d^2 D}{n^2 b} \left(\frac{m^2}{a^2} + \frac{n^2}{d^2} \right)^2 \rightarrow \frac{\pi^2 D}{d^2 b} \quad (22)$$

Therefore

$$\sigma_x + 4\sigma_y = \frac{4\pi^2 D}{d^2 b} \quad (23)$$

and

$$\sigma_x \equiv \sigma_\phi = K_{\phi P}$$

$$\sigma_y \equiv \sigma_r = K_{rP}$$

thus

$$P(K_\phi + 4K_r) = \frac{4\pi^2 D}{d^2 b}$$

$$P_c = \frac{\frac{4\pi^2 D}{d^2 b}}{(K_\phi + 4K_r)} \quad (24)$$

where

$$d = h$$

$$D = \frac{Eb^3}{12(1-\nu^2)}$$

thus

$$P_c = \frac{4\pi^2 E (b^2/h^2)}{12 (1 - \nu^2) (K_\phi + 4K_r)} \quad (25)$$

*Reference (2), (4), (8), and (22).

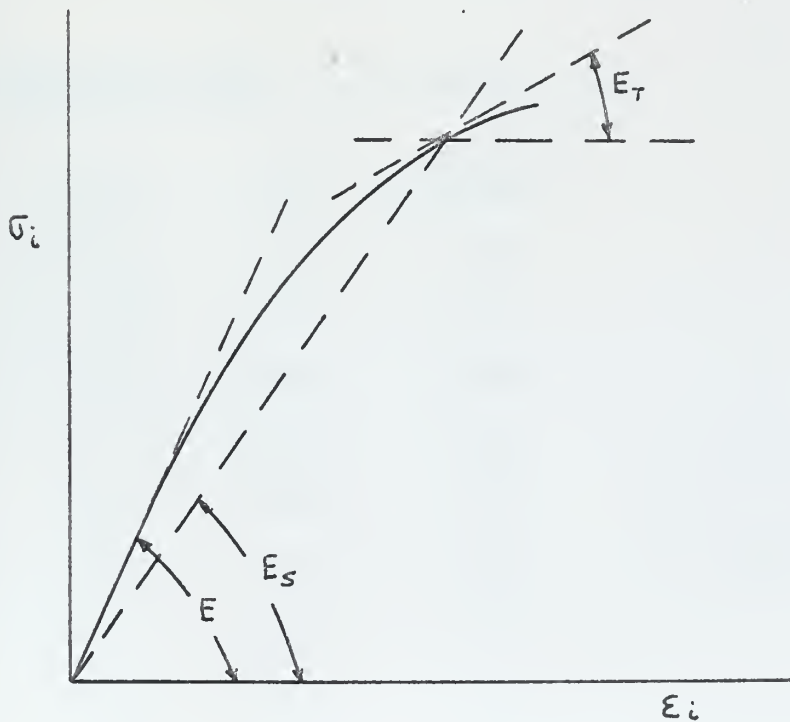
APPENDIX C

MATERIAL PROPERTIES*

The properties for the four materials utilized in the optimization studies are tabled below. The secant and tangent moduli are defined as

$$E_s = \frac{\sigma_i}{\epsilon_i}$$

$$E_T = \frac{d\sigma_i}{d\epsilon_i}$$



(1) HIGH STRENGTH STEEL ($\sigma_Y = 160,000$ psi)

σ_i/σ_Y	E_T/E	$\sqrt{E_S E_T}/E^2$	
0.65	1.00	1.00	
0.70	1.00	1.00	
0.75	1.00	1.00	
0.80	1.00	1.00	$E = 30 \times 10^6$ psi
0.85	0.95	0.98	$\nu_E = 0.30$
0.90	0.76	0.86	$\rho = 490$ pcf
0.95	0.55	0.72	
1.00	0.30	0.52	
1.05	0.12	0.30	
1.10	0.06	0.18	

(2) TITANIUM ALLOY ($\sigma_Y = 110,000$ psi)

σ_i/σ_Y	E_T/E	$\sqrt{E_S E_T}/E^2$	
0.65	1.00	1.00	
0.70	1.00	1.00	
0.75	0.85	0.98	
0.80	0.68	0.85	$E = 18 \times 10^6$ psi
0.85	0.55	0.75	$\nu_E = 0.30$
0.90	0.44	0.62	$\rho = 276$ pcf
0.95	0.32	0.50	
1.00	0.22	0.41	
1.05	0.15	0.30	
1.10	0.06	0.22	

(3) ALUMINUM ALLOY ($\sigma_Y = 65,000$)

σ_i/σ_Y	E_T/E	$\sqrt{E_S E_T/E^2}$
0.65	1.00	1.00
0.70	1.00	1.00
0.75	1.00	1.00
0.80	1.00	1.00
0.85	1.00	1.00
0.90	0.62	0.80
0.95	0.34	0.55
1.00	0.14	0.28
1.05	0.05	0.12
1.10	0.03	0.06

$$E = 10.8 \times 10^6 \text{ psi}$$

$$\nu_E = 0.30$$

$$\rho = 173 \text{ pcf}$$

(4) HY80 STEEL ($\sigma_Y = 80,000 \text{ psi}$)

σ_i/σ_Y	E_T/E	$\sqrt{E_S E_T/E^2}$
0.65	1.00	1.00
1.00	1.00	1.00
1.05	0.00	0.00

$$E = 30 \times 10^6 \text{ psi}$$

$$\nu_E = 0.30$$

$$\rho = 490 \text{ pcf}$$

APPENDIX D

SAMPLE OUTPUT OF COMPUTER PROGRAMS

1. STAS - Stress Analysis Program
2. FMCS - Failure Mode Criteria Program
3. OPTI - Optimization Program

STRESS ANALYSIS OF A WEB STIFFENED SANDWICH

CYLINDRICAL PRESSURE HL

OUTSIDE RADIUS(IN.)= 61.50
 INSIDE RADIUS(IN)= 58.50
 OUTSIDE THICKNESS (IN)= 0.60
 INSIDE THICKNESS= 0.60
 WEB THICKNESS(IN)= 0.60
 WEB SPACING(IN)= 5.40

MX00	MPHI00	MX01	MPHI01	MX0I	MPHIIO	MXII	MPHIII
-37.78	-47.87	-14.75	-40.96	-16.95	-41.91	-35.58	-47.50
FX00	FPHI00	FX0I	FPHI0I	FXIO	FPHIIO	FXII	FPHIII
-3.19	-36.45	-49.35	-50.30	-44.95	-51.19	-7.58	-39.98
HRW	HS	HTS	BTS	IEFF	D		
0.050	0.500	5.000	1.000	16.9	3.600		
WB=	0.377						
KR=	3.071	KP=	83.887				

STRESS ANALYSIS OF A WEB STIFFENED SANDWICH

CYLINDRICAL PRESSURE HL

OUTSIDE RADIUS(IN.)= 123.00
 INSIDE RADIUS(IN)= 117.00
 OUTSIDE THICKNESS (IN)= 1.20
 INSIDE THICKNESS= 1.20
 WEB THICKNESS(IN)= 1.20
 WEB SPACING(IN)= 10.80

MX00	MPHI00	MX01	MPHI01	MX0I	MPHIIO	MXII	MPHIII
-37.78	-47.87	-14.75	-40.96	-16.95	-41.91	-35.58	-47.50
FX00	FPHI00	FX0I	FPHI0I	FXIO	FPHIIO	FXII	FPHIII
-3.19	-36.45	-49.35	-50.30	-44.95	-51.19	-7.58	-39.98
HRW	HS	HTS	BTS	IEFF	D		
0.050	0.500	5.000	1.000	270.6	7.200		
WB=	0.377						
KR=	3.071	KP=	83.887				

STRESS ANALYSIS OF A WEB STIFFENED SANDWICH

CYLINDRICAL PRESSURE HL

OUTSIDE RADIUS(IN.)= 184.50
 INSIDE RADIUS(IN)= 175.50
 OUTSIDE THICKNESS (IN)= 1.80

INSIDE THICKNESS= 1.80
 WEB THICKNESS(IN)= 1.80
 WEB SPACING(IN)= 16.20

MX00	MPHI00	MX01	MPHI01	MX0I	MPHI10	MX11	MPHI11
-37.78	-47.87	-14.75	-40.96	-16.95	-41.91	-35.58	-47.50
FX00	FPHI00	FX01	FPHI01	Fx10	FPHI10	FX11	FPHI11
-3.19	-36.45	-49.35	-50.30	-44.95	-51.19	-7.58	-39.98
HRW	HS	HTS	BTS	IEFF	D		
0.050	0.500	5.000	1.000	1369.9	10.800		
WB=	0.377						
KR=	3.071	KP=	83.887				

COMPILE TIME= 7.53 SEC.EXECUTION TIME= 0.86 SEC.OBJECT CODE= 9856 BYTES

P	PF	FPC1	PPC2	PPC3	PPC4	PPC5	SIGMAI
STATE OF STRESS= 1							
2379.5	2087.0	9481.2	8365.5	10519.8	1423.8	45113.7	103999.90
2562.5	2247.6	9481.2	8365.5	10519.8	1423.8	45113.7	111999.90
2745.5	2408.1	9481.2	8365.5	10519.8	1423.8	45113.7	120000.00
2928.6	2568.6	9481.2	8365.5	10519.8	1423.8	45113.7	128000.00
3111.6	2729.2	9408.5	8464.1	9996.3	1362.1	42796.4	136000.00
3294.6	2889.7	8878.8	8237.6	8003.2	1089.7	34405.3	143999.90
3477.7	3050.3	7921.3	8071.0	5759.7	789.6	25005.6	151999.90
3660.7	3210.8	6568.5	7835.2	3173.9	430.1	13718.5	160000.00
3843.7	3371.3	5538.8	6712.7	1277.0	172.1	5614.1	168000.00
4026.8	3531.9	4049.4	5020.1	641.0	86.0	2910.4	176000.00

STATE OF STRESS= 2							
2429.7	2150.0	10136.7	8993.9	10505.1	1423.8	45113.7	103999.90
2616.6	2315.3	10136.7	8993.9	10505.1	1423.8	45113.7	111999.90
2803.5	2480.7	10136.7	8993.9	10505.1	1423.8	45113.7	120000.00
2990.4	2646.1	10136.7	8993.9	10505.1	1423.8	45113.7	128000.00
3177.3	2811.5	10057.1	9086.0	9981.9	1362.1	42796.4	136000.00
3364.2	2976.9	9271.2	8807.2	7990.6	1089.7	34405.3	143999.90
3551.1	3142.3	8453.6	8586.2	5789.2	788.6	25005.6	151999.90
3738.0	3307.6	7424.4	8283.3	3166.2	430.1	13718.5	160000.00
3924.9	3473.0	5891.8	7066.4	1272.6	172.1	5614.1	168000.00
4111.8	3638.4	4305.1	5278.3	638.3	86.0	2910.4	176000.00

PMS= 1941.8
PND= 3200.C

270.60 7.20 10.80 0.30 720.00 16000.C C.30E C8 1.20
GRAPHICAL DATA FOR FAILURE MODES

P	PF	PPC1	PPC2	PPC3	PPC4	PPC5	SIGMAI
STATE CF STRESS= 1							
2379.5	2087.0	5481.2	8365.4	10521.4	1434.0	45113.7	103999.90
2562.5	2247.6	5481.2	8365.4	10521.4	1434.0	45113.7	111999.90
2745.5	2408.1	9481.2	8365.4	10521.4	1434.0	45113.7	120000.00
2928.6	2568.6	5481.2	8365.4	10521.4	1434.0	45113.7	128000.00
3111.6	2729.2	9408.5	8464.1	9997.7	1262.3	42796.4	136000.00
3294.6	2889.7	8678.8	8237.6	8004.4	1089.9	34405.3	143999.90
3477.7	3050.3	7921.3	8071.0	5800.6	788.7	25005.6	151999.90
3660.7	3210.8	6968.5	7835.2	3174.2	420.2	13718.5	160000.00
3843.7	3371.3	5538.8	6712.7	1277.2	172.1	5614.1	168000.00
4026.8	3531.9	4049.4	5020.1	641.1	86.0	2910.4	176000.00

STATE OF STRESS= 2

2429.7	2150.0	10136.7	8993.9	10506.7	1424.0	45113.7	103999.90
2616.6	2315.3	10136.7	8993.9	10506.7	1434.0	45113.7	111999.90
2803.5	2480.7	10136.7	8993.9	10506.7	1434.0	45113.7	120000.00
2990.4	2646.1	10136.7	8993.9	10506.7	1424.0	45113.7	128000.00
3177.3	2811.5	10057.1	5086.0	9983.3	1362.3	42796.4	136000.00
3364.2	2976.9	9271.1	8807.2	7991.8	1089.9	34405.3	143999.90
3551.1	3142.3	8453.6	8586.2	5790.0	788.7	25005.6	151999.90
3738.0	3307.6	7424.4	8283.3	3166.7	430.2	13718.5	160000.00
3924.9	3473.0	5891.8	7066.4	1272.8	172.1	5614.1	168000.00

4111.6 3638.4 4305.1 5278.3 638.4 86.0 2910.4 176000.00
 PWS= 1541.8
 PND= 3200.0

1365.94 1C.80 16.20 0.30 1080.00 160000. 0.30E 08 1.80
 GRAPHICAL DATA FOR FAILURE MODES

P	PF	PPC1	PPC2	PPC3	PPC4	PPC5	SIGMAI
STATE OF STRESS= 1							
2379.5	2087.0	5481.2	8365.4	10521.6	1434.1	45113.7	103999.90
2562.5	2247.6	9481.2	8365.4	10521.6	1434.1	45112.7	111599.90
2745.5	2408.1	9481.2	8365.4	10521.6	1434.1	45113.7	120000.00
2928.6	2568.6	5481.2	8365.4	10521.6	1434.1	45113.7	128000.00
3111.6	2729.2	5408.5	8464.1	9997.9	1362.4	42796.5	136000.00
3254.6	2889.7	8678.8	8237.6	8004.5	1089.9	34405.3	143999.90
3477.7	3050.3	7921.3	8071.0	5800.7	768.7	25005.6	151999.90
3660.7	3210.8	6968.5	7835.2	3174.4	420.2	13718.5	160000.00
3843.7	3371.3	5538.8	6712.7	1277.3	172.1	5614.1	168000.00
4026.6	3531.9	4049.4	5020.1	641.1	66.0	2910.4	176000.00

STATE OF STRESS= 2							
2429.7	2150.0	10136.7	8993.9	10506.9	1434.1	45112.7	103999.90
2616.6	2315.3	10136.7	8993.9	10506.9	1434.1	45112.7	111599.90
2803.5	2480.7	10136.7	8993.9	10506.9	1434.1	45113.7	120000.00
2990.4	2646.1	10136.7	8993.9	10506.9	1434.1	45113.7	128000.00
3177.3	2811.5	10057.1	9085.9	9983.5	1362.4	42796.5	136000.00
3364.2	2976.9	5271.1	8807.2	7991.9	1089.9	34405.3	143999.90
3551.1	3142.3	8453.6	8586.2	5790.1	788.7	25005.6	151599.90
3738.0	3307.6	7424.4	8283.3	3166.8	430.2	13718.5	160000.00

3924.5	3473.0	5891.8	7066.4	1272.9	172.1	5614.1	168000.00
4111.8	3630.4	4305.1	5278.2	638.4	86.0	2510.4	176000.00
PWS=	1941.8						
PND=	3200.0						

COMPILE TIME= 4.78 SEC, EXECUTION TIME= 4.70 SEC, OBJECT CODE= 5048 BYTES, ARRAY AREA= 188 BYTES, I

56.000 0.105 0.105 0.113 57.143 6.897 1.143 1250.614 0.271
 56.000 0.105 0.105 0.113 57.143 6.742 1.233 1255.489 0.301
 56.000 0.105 0.105 0.113 57.143 5.357 1.143 1195.530 0.222
 56.000 0.105 0.105 0.113 57.143 5.263 1.333 1210.154 0.247
 56.000 0.105 0.105 0.113 57.143 4.380 1.143 1130.773 0.192
 56.000 0.105 0.105 0.113 57.143 4.317 1.333 1149.998 0.212
 WEIGHT/BUOYANCY= 0.125

COLLAPSE PRESSURE= 1000.
 DENSITY= 490.0
 NUC= C.30
 SIGNAY= 160000.
 INSIDE RADIUS= 50.000
 RRC YTO TTI IRW HTS HS BTS PF WD

54.000	C.160	0.140	0.077	26.667	1.869	0.933	1279.165	0.155
54.000	C.160	0.140	0.077	26.667	1.852	1.067	1312.820	0.165
54.000	C.160	0.150	0.077	25.806	1.869	0.503	1327.074	0.158
54.000	C.160	0.160	0.077	25.806	1.852	1.032	1361.894	0.168
54.000	C.160	0.160	0.077	25.000	1.869	0.675	1373.790	0.161
54.000	C.160	0.160	0.077	25.000	1.852	1.600	1409.727	0.170
55.000	C.160	0.150	0.095	32.258	2.315	1.632	1354.540	0.183
55.000	C.160	0.160	0.095	31.250	2.315	1.000	1443.640	0.185
WEIGHT/BUOYANCY=		0.155						

COMPILE TIME= 9.29 SEC. EXECUTION TIME= 419.42 SEC. OBJECT CODE= 14752 BYTES, ARRAY AREA= 128 B

Thesis

K8674 Kroner

118361

Stress analysis,
buckling analysis and
optimum proportions of
an isotropic web-
stiffened sandwich
cylindrical shell under
hydrostatic pressure.

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Thesis

K8674 Kroner

118361

Stress analysis,
buckling analysis and
optimum proportions of
an isotropic web-
stiffened sandwich
cylindrical shell under
hydrostatic pressure.

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Stress analysis, buckling analysis and o



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